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A TREATISE

ON

CHEMISTRY AND CHEMICAL ANALYSIS

PREPARED FOR STUDENTS OF
THE INTERNATIONAL CORRESPONDENCE SCHOOLS
SCRANTON, PA.

Volume VI

ANSWERS TO QUESTIONS

First Edition

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A KEY

TO ALL THE

QUESTIONS AND EXAMPLES

CONTAINED IN THE

QUESTION PAPERS INCLUDED IN VOLS. I–V.

It will be noticed that the Keys have been given the same section numbers as the Question Papers to which they refer. All article references refer to the Instruction Paper bearing the same section number as the Key in which it occurs, unless the title of some other Instruction Paper is given in connection with the article number.

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ARITHMETIC.

(QUESTIONS 1-75. SEC. 1.)

(1) See Art. **1.**

(2) See Art. **3.**

(3) See Arts. **5** and **6.**

(4) See Arts. **10** and **11.**

(5) 980 = Nine hundred eighty.
605 = Six hundred five.
28,284 = Twenty-eight thousand two hundred eighty-four.
9,006,042 = Nine million six thousand forty-two.
850,317,002 = Eight hundred fifty million three hundred seventeen thousand two.
700,004 = Seven hundred thousand four.

(6) Seven thousand six hundred = 7,600.
Eighty-one thousand four hundred two = 81,402.
Five million four thousand seven = 5,004,007.
One hundred eight million ten thousand one = 108,010,001.
Eighteen million six = 18,000,006.
Thirty thousand ten = 30,010.

$$\begin{array}{r} 3\,2\,9\,0 \\ 5\,0\,4 \\ 8\,6\,5\,4\,0\,3 \\ 2\,0\,7\,4 \\ 8\,1 \\ 7 \\ \hline 8\,7\,1\,3\,5\,9 \end{array} \quad \text{Ans.}$$

(7) In adding whole numbers, place the numbers to be added directly under each other so that the extreme right-hand figures will stand in the same column, regardless of the position of those at the left. Add the first column of figures at the extreme right, which equals 19 units, or 1 ten and 9 units. We place

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9 units under the units column, and reserve 1 ten for the column of tens. $1+8+7+9 = 25$ tens, or 2 hundreds and 5 tens. Place 5 tens under the tens column, and reserve 2 hundreds for the hundreds column. $2+4+5+2 = 13$ hundreds, or 1 thousand and 3 hundreds. Place 3 hundreds under the hundreds column, and reserve the 1 thousand for the thousands column. $1+2+5+3 = 11$ thousands, or 1 ten thousand and 1 thousand. Place the 1 thousand in the column of thousands, and reserve the 1 ten thousand for the column of ten thousands. $1+6 = 7$ ten thousands. Place this 7 ten thousands in the ten thousands column. There is but one figure, 8, in the hundreds of thousands place in the numbers to be added, so it is placed in the hundreds of thousands column of the sum.

A simple (though less scientific) explanation of the same problem is the following: $7+1+4+3+4+0 = 19$; write the 9 and reserve the 1. $1+8+7+0+0+9 = 25$; write the 5 and reserve the 2. $2+0+4+5+2 = 13$; write the 3 and reserve the 1. $1+2+5+3 = 11$; write the 1 and reserve 1. $1+6 = 7$; write the 7. Bring down the 8 to its place in the sum.

(8)

$$\begin{array}{r}
709 \\
8304725 \\
391 \\
100302 \\
300 \\
909 \\
\hline
8407336
\end{array} \quad \text{Ans.}$$

(9) (*a*) In subtracting whole numbers, place the subtrahend, or smaller number, under the minuend, or larger number, so that the right-hand figures stand directly under each other. Begin at the *right* to subtract. We cannot subtract 8 units from 2 units, so we take 1 ten from the 6 tens and add it to the 2 units. 1 ten $=$ 10 units, so we have 10 units $+$ 2 units $=$ 12 units. Then 8 units from 12 units leaves 4 units. We took 1 ten from 6 tens, so only 5

tens remain. 3 tens from 5 tens leaves 2 tens. In the hundreds column we have 3 hundreds from 9 hundreds leaves 6 hundreds. We cannot subtract 3 thousands from 0 thousands, so we take 1 ten thousand from 5 ten thousands and add it to the 0 thousands. 1 ten thousand = 10 thousands, and 10 thousands + 0 thousands = 10 thousands. Subtracting, we have 3 thousands from 10 thousands leaves 7 thousands. We took 1 ten thousand from 5 ten thousands and have 4 ten thousands remaining. Since there are no ten thousands in the subtrahend, the 4 in the ten thousands column in the minuend is brought down into the same column in the remainder, because 0 from 4 leaves 4.

$$\begin{array}{r}50962\\ 3338\\ \hline 47624\end{array}\quad \text{Ans.}$$

$$(b)\quad \begin{array}{r}15339\\ 10001\\ \hline 5338\end{array}\quad \text{Ans.}$$

(10) $$(a)\quad \begin{array}{r}70968\\ 32975\\ \hline 37993\end{array}\quad \text{Ans.} \qquad (b)\quad \begin{array}{r}100000\\ 98735\\ \hline 1265\end{array}\quad \text{Ans.}$$

(11) We have given the minuend or greater number (1,004) and the difference or remainder (49). Placing these in the usual form of subtraction, we have $\begin{array}{r}1004\\ \text{——}\\ \hline 49\end{array}$ in which the dash (——) represents the number sought. This number is evidently *less* than 1,004 by the difference 49, hence, 1,004 − 49 = 955, the smaller number. For the sum of the two numbers we then have

$$\begin{array}{rl}1004 & \textit{larger}\\ 955 & \textit{smaller}\\ \hline 1959 & \textit{sum.}\end{array}\quad \text{Ans.}$$

Or, this problem may be solved as follows: If the greater of two numbers is 1,004, and the difference between them is 49, then it is evident that the smaller number must be equal to the difference between the greater number (1,004) and the difference (49); or, 1,004 − 49 = 955, the smaller number. Since the greater number equals 1,004 and the smaller number equals 955, their sum equals 1,004 + 955 = 1,959. Ans.

(12) The numbers connected by the plus (+) sign must first be added. Performing these operations we have

$$\begin{array}{r} 5962 \\ 8471 \\ 9023 \\ \hline 23456 \text{ } \textit{sum.} \end{array} \qquad\qquad \begin{array}{r} 3874 \\ 2039 \\ \hline 5913 \text{ } \textit{sum.} \end{array}$$

Subtracting the smaller number (5,913) from the greater (23,456), we have

$$\begin{array}{r} 23456 \\ 5913 \\ \hline 17543 \text{ } \textit{difference.} \quad \text{Ans.} \end{array}$$

(13)

$$\begin{array}{rl} \$44675 & = \text{amount willed to his son.} \\ 26380 & = \text{amount willed to his daughter.} \\ \hline \$71055 & = \text{amount willed to his two children.} \\ \$125000 & = \text{amount willed to his wife and two children.} \\ 71055 & = \text{amount willed to his two children.} \\ \hline \$53945 & = \text{amount willed to his wife. Ans.} \end{array}$$

(14) In the multiplication of whole numbers, place the multiplier under the multiplicand, and multiply each term of the multiplicand by each term of the multiplier, writing the right-hand figure of each product obtained under the term of the multiplier which produces it.

(*a*)

$$\begin{array}{r} 526387 \\ 7 \\ \hline 3684709 \text{ Ans.} \end{array}$$

7 times 7 units = 49 units, or 4 tens and 9 units. We write the 9 units and reserve the 4 tens. 7 times 8 tens = 56 tens; 56 + 4 tens reserved = 60 tens, or 6 hundreds and 0 tens. Write the 0 tens and reserve the 6 hundreds. 7 × 3 hundreds = 21 hundreds; 21 + 6 hundreds reserved = 27 hundreds, or 2 thousands and 7 hundreds. Write the 7 hundreds and reserve the 2 thousands. 7 × 6 thousands = 42 thousands;

42 + 2 thousands reserved = 44 thousands, or 4 ten thousands and 4 thousands. Write the 4 thousands and reserve the 4 ten thousands. 7 × 2 ten thousands = 14 ten thousands; 14 + 4 ten thousands reserved = 18 ten thousands, or 1 hundred thousand and 8 ten thousands. Write the 8 ten thousands and reserve the 1 hundred thousand. 7 × 5 hundred thousands = 35 hundred thousands; 35 + 1 hundred thousand reserved = 36 hundred thousands. Since there are no more figures in the multiplicand to be multiplied, we write the 36 hundred thousands in the product. This completes the multiplication.

A simpler (though less scientific) explanation of the same problem is the following:

7 times 7 = 49; write the 9 and reserve the 4. 7 times 8 = 56; 56 + 4 reserved = 60; write the 0 and reserve the 6. 7 times 3 = 21; 21 + 6 reserved = 27; write the 7 and reserve the 2. 7 × 6 = 42; 42 + 2 reserved = 44; write the 4 and reserve 4. 7 × 2 = 14; 14 + 4 reserved = 18; write the 8 and reserve the 1. 7 × 5 = 35; 35 + 1 reserved = 36; write the 36.

In this case the multiplier is 17 *units*, or 1 ten and 7 units, so that the product is obtained by adding two partial products, namely, 7 × 700,298 and 10 × 700,298. The actual operation is performed as follows:

$$
(b)\qquad
\begin{array}{r}
700298\\
17\\
\hline
4902086\\
700298\\
\hline
11905066
\end{array}\ \text{Ans.}
$$

7 times 8 = 56; write the 6 and reserve the 5. 7 times 9 = 63; 63 + 5 reserved = 68; write the 8 and reserve the 6. 7 times 2 = 14; 14 + 6 reserved = 20; write the 0 and reserve the 2. 7 times 0 = 0; 0 + 2 reserved = 2; write the 2. 7 times 0 = 0; 0 + 0 reserved = 0; write the 0. 7 times 7 = 49; 49 + 0 reserved = 49; write the 49.

To multiply by the 1 ten we say 1 times 700,298 = 700,298, and write 700,298 under the first partial product, as shown, with the right-hand figure 8 under the multiplier 1. Add the two partial products; their sum equals the entire product.

(*c*) Multiply any two of the numbers together and multiply their product by the third number.

$$\begin{array}{r l}
217 & \\
\underline{103} & \\
651 & \\
\underline{2170} & \\
22351 & \\
\underline{67} & \\
156457 & \\
\underline{134106} & \\
1497517 & \text{Ans.}
\end{array}$$

(15) If your watch ticks every second, then to find how many times it ticks in 1 week, it is necessary to find the number of seconds in one week.

$$\begin{array}{r l}
60 & \text{seconds} = 1 \text{ minute.}\\
\underline{60} & \text{minutes} = 1 \text{ hour.}\\
3600 & \text{seconds} = 1 \text{ hour.}\\
\underline{24} & \text{hours} = 1 \text{ day.}\\
14400 & \\
\underline{7200} & \\
86400 & \text{seconds} = 1 \text{ day.}\\
\underline{7} & \text{days} = 1 \text{ week.}\\
604800 & \text{seconds in 1 week, or the number of times that}\\
 & \text{your watch ticks in 1 week. Ans.}
\end{array}$$

(16) If a monthly publication contains 24 pages, a yearly volume will contain 12×24, or 288 pages, since there are 12 months in one year; and eight yearly volumes will contain 8×288, or 2,304 pages.

$$\begin{array}{r l}
24 & \\
\underline{12} & \\
288 & \\
\underline{8} & \\
2304 & \text{Ans.}
\end{array}$$

(17) If an engine and boiler are worth \$3,246, and the building is worth 3 times as much, plus \$1,200, then the building is worth

$$\begin{array}{r r l}
 & \$3246 & \\
\times & \underline{3} & \\
 & 9738 & \\
+ & \underline{1200} & \\
 & \$10938 & = \text{value of building.}
\end{array}$$

If the tools are worth twice as much as the building, plus \$1,875, then the tools are worth

$$\begin{array}{rl}
\$\,1\,0\,9\,3\,8 & \\
\times \qquad\quad 2 & \\
\cline{1-1}
2\,1\,8\,7\,6 & \\
+ \quad 1\,8\,7\,5 & \\
\cline{1-1}
\$\,2\,3\,7\,5\,1 & = \text{value of tools.}
\end{array}$$

$$\begin{array}{lrl}
\text{Value of building} & = \$\,1\,0\,9\,3\,8 & \\
\text{Value of tools} & = \quad 2\,3\,7\,5\,1 & \\
\cline{2-2}
 & \$\,3\,4\,6\,8\,9 & = \text{value of the building and tools. } (a) \text{ Ans.} \\
\text{Value of engine and boiler} & = \$ \quad 3\,2\,4\,6 & \\
\text{Value of building and tools} & = \quad 3\,4\,6\,8\,9 & \\
\cline{2-2}
 & \$\,3\,7\,9\,3\,5 & = \text{value of the whole plant. } (b) \text{ Ans.}
\end{array}$$

(18) (*a*) $(72 \times 48 \times 28 \times 5) \div (96 \times 15 \times 7 \times 6)$.

Placing the numerator over the denominator the problem becomes

$$\frac{72 \times 48 \times 28 \times 5}{96 \times 15 \times 7 \times 6} = ?$$

The 5 in the dividend and 15 in the divisor are both divisible by 5, since 5 divided by 5 equals 1, and 15 divided by 5 equals 3. Cross off the 5 and write the 1 over it; also, cross off the 15 and write the 3 under it. Thus,

$$\frac{72 \times 48 \times 28 \times \overset{1}{\cancel{5}}}{96 \times \underset{3}{\cancel{15}} \times 7 \times 6} =$$

The 5 and 15 are not to be considered any longer, and, in fact, may be erased entirely and the 1 and 3 placed in their stead, and treated as if the 5 and the 15 never existed. Thus,

$$\frac{72 \times 48 \times 28 \times 1}{96 \times 3 \times 7 \times 6} =$$

72 in the dividend and 96 in the divisor are divisible by 12, since 72 divided by 12 equals 6, and 96 divided by 12 equals 8. Cross off the 72 and write the 6 over it; also, cross off the 96 and write the 8 under it. Thus,

$$\frac{\overset{6}{\cancel{72}}\times 48\times 28\times 1}{\underset{8}{\cancel{96}}\times 3\times 7\times 6}=$$

The 72 and 96 are not to be considered any longer, and, in fact, may be erased entirely and the 6 and 8 placed in their stead, and treated as if the 72 and 96 never existed. Thus,

$$\frac{6\times 48\times 28\times 1}{8\times 3\times 7\times 6}=$$

Again, 28 in the dividend and 7 in the divisor are divisible by 7, since 28 divided by 7 equals 4, and 7 divided by 7 equals 1. Cross off the 28 and write the 4 over it; also, cross off the 7 and write the 1 under it. Thus,

$$\frac{6\times 48\times \overset{4}{\cancel{28}}\times 1}{8\times 3\times \underset{1}{\cancel{7}}\times 6}=$$

The 28 and 7 are not to be considered any longer, and, in fact, may be erased entirely and the 4 and 1 placed in their stead, and treated as if the 28 and 7 never existed. Thus,

$$\frac{6\times 48\times 4\times 1}{8\times 3\times 1\times 6}=$$

Again, 48 in the dividend and 6 in the divisor are divisible by 6, since 48 divided by 6 equals 8, and 6 divided by 6 equals 1. Cross off the 48 and write the 8 over it; also, cross off the 6 and write the 1 under it. Thus,

$$\frac{6\times \overset{8}{\cancel{48}}\times 4\times 1}{8\times 3\times 1\times \underset{1}{\cancel{6}}}=$$

The 48 and 6 are not to be considered any longer, and, in fact, may be erased entirely and the 8 and 1 placed in their stead, and treated as if the 48 and 6 never existed. Thus,

$$\frac{6\times8\times4\times1}{8\times3\times1\times1}=$$

Again, 6 in the dividend and 3 in the divisor are divisible by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. Cross off the 6 and write the 2 over it; also, cross off the 3 and write the 1 under it. Thus,

$$\frac{\overset{2}{\cancel{6}}\times8\times4\times1}{8\times\underset{1}{\cancel{3}}\times1\times1}=$$

The 6 and 3 are not to be considered any longer, and, in fact, may be erased entirely and the 2 and 1 placed in their stead, and treated as if the 6 and 3 never existed. Thus,

$$\frac{2\times8\times4\times1}{8\times1\times1\times1}=$$

Canceling the 8 in the dividend and the 8 in the divisor, the result is

$$\frac{2\times\overset{1}{\cancel{8}}\times4\times1}{\underset{1}{\cancel{8}}\times1\times1\times1}=\frac{2\times1\times4\times1}{1\times1\times1\times1}.$$

Since there are no two remaining numbers (one in the dividend and one in the divisor) divisible by any number except 1, without a remainder, it is impossible to cancel further.

Multiply all the uncanceled numbers in the dividend together, and divide their product by the product of all the uncanceled numbers in the divisor. The result will be the quotient. The product of all the uncanceled numbers in the dividend equals $2\times1\times4\times1=8$; the product of all the uncanceled numbers in the divisor equals $1 \times 1 \times 1 \times 1 = 1$.

Hence, $$\frac{2 \times 1 \times 4 \times 1}{1 \times 1 \times 1 \times 1} = \frac{8}{1} = 8. \quad \text{Ans.}$$

Or, $$\frac{\overset{\overset{2}{\cancel{6}}}{\cancel{72}} \times \overset{\overset{1}{\cancel{8}}}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \overset{1}{\cancel{5}}}{\underset{\underset{1}{\cancel{8}}}{\cancel{96}} \times \underset{\underset{1}{\cancel{3}}}{\cancel{15}} \times \underset{1}{\cancel{7}} \times \underset{1}{\cancel{6}}} = \frac{8}{1} = 8. \quad \text{Ans.}$$

(*b*) $(80 \times 60 \times 50 \times 16 \times 14) \div (70 \times 50 \times 24 \times 20)$.

Placing the numerator over the denominator, the problem becomes

$$\frac{80 \times 60 \times 50 \times 16 \times 14}{70 \times 50 \times 24 \times 20} = ?$$

The 50 in the dividend and 70 in the divisor are both divisible by 10, since 50 divided by 10 equals 5, and 70 divided by 10 equals 7. Cross off the 50 and write the 5 over it; also, cross off the 70 and write the 7 under it. Thus,

$$\frac{80 \times 60 \times \overset{5}{\cancel{50}} \times 16 \times 14}{\underset{7}{\cancel{70}} \times 50 \times 24 \times 20} =$$

The 50 and 70 are not to be considered any longer, and, in fact, may be erased entirely and the 5 and 7 placed in their stead, and treated as if the 50 and 70 never existed. Thus,

$$\frac{80 \times 60 \times 5 \times 16 \times 14}{7 \times 50 \times 24 \times 20} =$$

Also, 80 in the dividend and 20 in the divisor are divisible by 20, since 80 divided by 20 equals 4, and 20 divided by 20 equals 1. Cross off the 80 and write the 4 over it; also, cross off the 20 and write the 1 under it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times 60 \times 5 \times 16 \times 14}{7 \times 50 \times 24 \times \underset{1}{\cancel{20}}} =$$

The 80 and 20 are not to be considered any longer, and, in fact, may be erased entirely and the 4 and 1 placed in their stead, and treated as if the 80 and 20 never existed. Thus,

$$\frac{4\times 60\times 5\times 16\times 14}{7\times 50\times 24\times 1}=$$

Again, 16 in the dividend and 24 in the divisor are divisible by 8, since 16 divided by 8 equals 2, and 24 divided by 8 equals 3. Cross off the 16 and write the 2 over it; also, cross off the 24 and write the 3 under it. Thus,

$$\frac{4\times 60\times 5\times \overset{2}{\cancel{16}}\times 14}{7\times 50\times \underset{3}{\cancel{24}}\times 1}=$$

The 16 and 24 are not to be considered any longer, and, in fact, may be erased entirely and the 2 and 3 placed in their stead, and treated as if the 16 and 24 never existed. Thus,

$$\frac{4\times 60\times 5\times 2\times 14}{7\times 50\times 3\times 1}=$$

Again, 60 in the dividend and 50 in the divisor are divisible by 10, since 60 divided by 10 equals 6, and 50 divided by 10 equals 5. Cross off the 60 and write the 6 over it; also, cross off the 50 and write the 5 under it. Thus,

$$\frac{4\times \overset{6}{\cancel{60}}\times 5\times 2\times 14}{7\times \underset{5}{\cancel{50}}\times 3\times 1}=$$

The 60 and 50 are not to be considered any longer, and, in fact, may be erased entirely and the 6 and 5 placed in their stead, and treated as if the 60 and 50 never existed. Thus,

$$\frac{4\times 6\times 5\times 2\times 14}{7\times 5\times 3\times 1}=$$

The 14 in the dividend and 7 in the divisor are divisible by 7, since 14 divided by 7 equals 2, and 7 divided by 7 equals 1.

Cross off the 14 and write the 2 over it; also, cross off the 7 and write the 1 under it. Thus,

$$\frac{4\times6\times5\times2\times\overset{2}{\cancel{14}}}{\underset{1}{\cancel{7}}\times5\times3\times1}=$$

The 14 and 7 are not to be considered any longer, and, in fact, may be erased entirely and the 2 and 1 placed in their stead, and treated as if the 14 and 7 never existed. Thus,

$$\frac{4\times6\times5\times2\times2}{1\times5\times3\times1}=$$

The 5 in the dividend and the 5 in the divisor are divisible by 5, since 5 divided by 5 equals 1. Cross off the 5 of the dividend and write the 1 over it; also, cross off the 5 of the divisor and write the 1 under it. Thus,

$$\frac{4\times6\times\overset{1}{\cancel{5}}\times2\times2}{1\times\underset{1}{\cancel{5}}\times3\times1}=$$

The 5 in the dividend and 5 in the divisor are not to be considered any longer, and, in fact, may be erased entirely and 1 and 1 placed in their stead, and treated as if the 5 and 5 never existed. Thus,

$$\frac{4\times6\times1\times2\times2}{1\times1\times3\times1}=$$

The 6 in the dividend and 3 in the divisor are divisible by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. Cross off the 6 and place the 2 over it; also, cross off the 3 and place the 1 under it. Thus,

$$\frac{4\times\overset{2}{\cancel{6}}\times1\times2\times2}{1\times1\times\underset{1}{\cancel{3}}\times1}=$$

The 6 and 3 are not to be considered any longer, and, in fact, may be erased entirely and 2 and 1 placed in their stead, and treated as if the 6 and 3 never existed. Thus,

$$\frac{4\times2\times1\times2\times2}{1\times1\times1\times1} = \frac{32}{1} = 32. \quad \text{Ans.}$$

Hence,

$$\frac{\overset{4}{\cancel{80}}\times\overset{\overset{2}{\cancel{6}}}{\cancel{60}}\times\overset{\overset{1}{\cancel{5}}}{\cancel{50}}\times\overset{2}{\cancel{16}}\times\overset{2}{\cancel{14}}}{\underset{\underset{1}{\cancel{7}}}{\cancel{70}}\times\underset{\underset{1}{\cancel{5}}}{\cancel{50}}\times\underset{\underset{1}{\cancel{3}}}{\cancel{24}}\times\underset{1}{\cancel{20}}} = \frac{4\times2\times1\times2\times2}{1\times1\times1\times1} = \frac{32}{1} = 32. \quad \text{Ans.}$$

(19) 28 acres of land at $133 an acre would cost

$$\begin{array}{r} 28\times\$\,1\,3\,3 = \$3{,}724 \\ \underline{2\,8} \\ 1\,0\,6\,4 \\ \underline{2\,6\,6} \\ \$\,3\,7\,2\,4 \end{array}$$

If a mechanic earns $1,500 a year and his expenses are $968 per year, then he would save $ 1 5 0 0 — $968, or $532 per year.

$$\begin{array}{r} \$\,1\,5\,0\,0 \\ \underline{9\,6\,8} \\ \$\,5\,3\,2 \end{array}$$

If he saves $532 in 1 year, to save $3,724 it would take as many years as $532 is contained times in $3,724, or 7 years.

$$\begin{array}{l} 5\,3\,2\,)\,3\,7\,2\,4\,(\,7 \text{ years.} \quad \text{Ans.} \\ \underline{\underline{3\,7\,2\,4}} \end{array}$$

(20) If the freight train ran 365 miles in one week, and 3 times as far lacking 246 miles the next week, then it ran (3×365 miles)—246 miles, or 849 miles the second week. Thus,

$$\begin{array}{rr} & 3\,6\,5 \\ & \underline{3} \\ & 1\,0\,9\,5 \\ & \underline{2\,4\,6} \\ \textit{difference} & 8\,4\,9 \text{ miles.} \quad \text{Ans.} \end{array}$$

(21) The distance from Philadelphia to Pittsburg is 354 miles. Since there are 5,280 feet in 1 mile, in 354 miles there are 354 × 5,280 feet, or 1,869,120 feet. If the driving wheel of the locomotive is 16 feet in circumference, then in going from Philadelphia to Pittsburg, a distance of 1,869,120 feet, it will make 1,869,120 ÷ 16, or 116,820 revolutions.

```
16)1869120(116820 rev.   Ans.
   16
   --
    26
    16
    --
    109
     96
    ---
     131
     128
     ---
       32
       32
       --
```

(22) (*a*)

```
576)589824(1024   Ans.
    576
    ----
     1382
     1152
     ----
      2304
      2304
      ----
```

(*b*)

```
43911)369730620(8420   Ans.
      351288
      ------
       184426
       175644
       ------
         87822
         87822
         -----
```

(*c*)

```
505)2527525(5005   Ans.
    2525
    -------
       2525
       2525
       ----
```

```
(d)   1234)4961794302(4020903 Ans.
           4936
           ────
             2579
             2468
             ────
              11143
              11106
              ─────
                 3702
                 3702
                 ────
```

(23) The harness evidently cost the difference between $444 and the amount which he paid for the horse and wagon. Since $264 + $153 = $417, the amount paid for the horse and wagon, $444 − $417 = $27, the cost of the harness.

```
     $264              $444
      153               417
     ────              ────
     $417              $ 27  Ans.
```

```
(24)  (a)              (b)                 (c)
     1024             5005               43911
      576              505                8420
     ────           ──────           ─────────
     6144            25025              878220
    7168            250250             175644
   5120             ───────           351288
   ──────           2527525  Ans.     ─────────
   589824  Ans.                       369730620  Ans.
```

(25) Since there are 12 months in a year, the number of days the man works is 25 × 12 = 300 days. As he works 10 hours each day, the number of hours that he works in one year is 300 × 10 = 3,000 hours. Hence, he receives for his work 3,000 × 30 = 90,000 cents, or 90,000 ÷ 100 = $900. Ans.

(26) See Art. **71.**

(27) See Art. **77.**

(28) See Art. **73.**

(29) See Art. **73.**

(30) See Art. **75.**

(31) $\frac{13}{8}$ is an improper fraction, since its numerator, 13, is greater than its denominator, 8.

(32) $4\frac{1}{2}$; $14\frac{3}{10}$; $85\frac{4}{19}$.

(33) To reduce a fraction to its lowest terms means to change its form without changing its value. In order to do this, we must divide both numerator and denominator by the same number until we can no longer find any number (except 1) which will divide both of these terms without a remainder.

To reduce the fraction $\frac{4}{8}$ to its lowest terms, we divide both numerator and denominator by 4, and obtain as a result the fraction $\frac{1}{2}$. Thus, $\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$; similarly, $\frac{4 \div 4}{16 \div 4} = \frac{1}{4}$; $\frac{8 \div 4}{32 \div 4} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$; $\frac{32 \div 8}{64 \div 8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$. Ans.

(34) When the denominator of any number is not expressed, it is understood to be 1, so that $\frac{6}{1}$ is the same as $6 \div 1$, or 6. To reduce $\frac{6}{1}$ to an improper fraction whose denominator is 4, we must multiply both numerator and denominator by some number which will make the denominator of 6 equal to 4. Since this denominator is 1, by multiplying both terms of $\frac{6}{1}$ by 4 we shall have $\frac{6 \times 4}{1 \times 4} = \frac{24}{4}$, which has the *same value* as 6, but has a *different form*. Ans.

(35) In order to reduce a mixed number to an improper fraction, we must multiply the whole number by the denominator of the fraction and add the numerator of the fraction to that product. This *result* is the numerator of the improper fraction, of which the denominator is the denominator of the fractional part of the mixed number.

$7\frac{7}{8}$ means the same as $7 + \frac{7}{8}$. In 1 there are $\frac{8}{8}$, hence in 7 there are $7 \times \frac{8}{8} = \frac{56}{8}$; $\frac{56}{8}$ plus the $\frac{7}{8}$ of the mixed number $= \frac{56}{8} + \frac{7}{8} = \frac{63}{8}$, which is the required improper fraction.

$$13\frac{5}{16} = \frac{(13 \times 16) + 5}{16} = \frac{213}{16};\ 10\frac{3}{4} = \frac{(10 \times 4) + 3}{4} = \frac{43}{4}.$$

(36) The value of a fraction is obtained by dividing the numerator by the denominator.

To obtain the value of the fraction $\frac{13}{2}$ we divide the numerator, 13, by the denominator, 2. 2 is contained in 13, 6 times, with 1 remaining. This 1 remaining is written over the denominator, 2, thereby making the fraction $\frac{1}{2}$, which is annexed to the whole number, 6, and we obtain $6\frac{1}{2}$ as the mixed number. The reason for performing this operation is the following: In 1 there are $\frac{2}{2}$ (two halves), and in $\frac{13}{2}$ (thirteen halves) there are as many ones (1) as 2 is contained times in 13, which is 6, and $\frac{1}{2}$ (one-half) remaining. Hence, $\frac{13}{2} = 6 + \frac{1}{2} = 6\frac{1}{2}$, the required mixed number.

$\frac{17}{4} = 4\frac{1}{4}$; $\frac{69}{16} = 4\frac{5}{16}$; $\frac{16}{8} = 2$; $\frac{67}{64} = 1\frac{3}{64}$. Ans.

(37) In division of fractions, invert the divisor (or, in other words, turn it upside down) and then proceed as in multiplication.

(*a*) $35 \div \frac{5}{16} = \frac{35}{1} \times \frac{16}{5} = \dfrac{35 \times 16}{1 \times 5} = \frac{560}{5} = 112.$ Ans.

(*b*) $\frac{9}{16} \div 3 = \frac{9}{16} \div \frac{3}{1} = \frac{9}{16} \times \frac{1}{3} = \dfrac{9 \times 1}{16 \times 3} = \frac{9}{48} = \frac{3}{16}.$ Ans.

(*c*) $\frac{17}{2} \div 9 = \frac{17}{2} \div \frac{9}{1} = \frac{17}{2} \times \frac{1}{9} = \dfrac{17 \times 1}{2 \times 9} = \frac{17}{18}.$ Ans.

(*d*) $\frac{113}{64} \div \frac{7}{16} = \frac{113}{64} \times \frac{16}{7} = \dfrac{113 \times 16}{64 \times 7} = \frac{1808}{448} = \frac{452}{112} = \frac{113}{28}.$

$\frac{113}{28}$

28) 1 1 3 ($4\frac{1}{28}$. Ans.

$\underline{1\ 1\ 2}$

1

(*e*) $15\frac{3}{4} \div 4\frac{3}{8} = ?$ Before proceeding with the division, reduce both of the mixed numbers to improper fractions. Thus, $15\frac{3}{4} = \dfrac{(15 \times 4) + 3}{4} = \dfrac{60 + 3}{4} = \frac{63}{4}$, and $4\frac{3}{8} = \dfrac{(4 \times 8) + 3}{8} = \dfrac{32 + 3}{8} = \frac{35}{8}$. The problem is now $\frac{63}{4} \div \frac{35}{8} = ?$ As before, invert the divisor and multiply; $\frac{63}{4} \div \frac{35}{8} = \frac{63}{4} \times \frac{8}{35} = \dfrac{63 \times 8}{4 \times 35} = \frac{504}{140} = \frac{252}{70} = \frac{126}{35} = \frac{18}{5}.$

$\frac{18}{5}$

5) 1 8 ($3\frac{3}{5}$. Ans.

$\underline{1\ 5}$

3

(38) $\frac{1}{8} + \frac{2}{8} + \frac{5}{8} = \dfrac{1 + 2 + 5}{8} = \frac{8}{8} = 1.$ Ans.

When the denominators of the fractions to be added are alike, we know that the units are divided into the same number of parts (in this case eighths); we, therefore, add the numerators of the fractions to find the number of parts (eighths) taken or considered, thereby obtaining $\frac{8}{8}$ or 1 as the sum.

(39) When the denominators are *not* alike we know that the units are divided into unequal parts, so before adding them we must find a common denominator for the denominators of all the fractions. Reduce the fractions to fractions having this common denominator, add the numerators, and write the sum over the common denominator.

In this case, the least common denominator, or the least number that will contain all the denominators, is 16; hence, we must reduce all these fractions to 16ths and then add their numerators.

$\frac{1}{4}+\frac{3}{8}+\frac{5}{16}=?$ To reduce the fraction $\frac{1}{4}$ to a fraction having 16 for a denominator, we must multiply both terms of the fraction by some number which will make the denominator 16. This number evidently is 4; hence, $\frac{1\times4}{4\times4}=\frac{4}{16}$.

Similarly, both terms of the fraction $\frac{3}{8}$ must be multiplied by 2 to make the denominator 16, and we have $\frac{3\times2}{8\times2}=\frac{6}{16}$.

The fractions now have a common denominator, 16; hence, we find their sum by adding the numerators and placing their sum over the common denominator, thus: $\frac{4}{16}+\frac{6}{16}+\frac{5}{16}=\frac{4+6+5}{16}=\frac{15}{16}$. Ans.

(40) When mixed numbers and whole numbers are to be added, add the fractional parts of the mixed numbers separately, and if the resulting fraction is an improper fraction, reduce it to a whole or mixed number. Next, add all the whole numbers, including the one obtained from the addition of the fractional parts, and annex to their sum the fraction of the mixed number obtained from reducing the improper fraction.

$42+31\frac{5}{8}+9\frac{7}{16}=?$ Reducing $\frac{5}{8}$ to a fraction having a denominator of 16, we have $\frac{5\times2}{8\times2}=\frac{10}{16}$. Adding the two fractional parts of the mixed numbers, we have $\frac{10}{16}+\frac{7}{16}$ $= \frac{10+7}{16} = \frac{17}{16} = 1\frac{1}{16}$.

The problem now becomes $42+31+9+1\frac{1}{16} = ?$

Adding all the whole numbers and the number obtained from adding the fractional parts of the mixed numbers, we obtain $83\frac{1}{16}$ as their sum.

$$\begin{array}{r} 42 \\ 31 \\ 9 \\ 1\frac{1}{16} \\ \hline 83\frac{1}{16} \end{array}$$ Ans.

(41) $29\frac{3}{4}+50\frac{5}{8}+41+69\frac{3}{16} = ?$ $\frac{3}{4} = \frac{3\times4}{4\times4} = \frac{12}{16}$. $\frac{5}{8} = \frac{5\times2}{8\times2} = \frac{10}{16}$. $\frac{12}{16}+\frac{10}{16}+\frac{3}{16} = \frac{12+10+3}{16} = \frac{25}{16} = 1\frac{9}{16}$.

The problem now becomes $29+50+41+69+1\frac{9}{16} = ?$

$$\begin{array}{rl} 29 & \text{square inches.} \\ 50 & \text{square inches.} \\ 41 & \text{square inches.} \\ 69 & \text{square inches.} \\ 1\frac{9}{16} & \text{square inches.} \\ \hline 190\frac{9}{16} & \text{square inches. Ans.} \end{array}$$

(42) (*a*) $\frac{7}{\frac{3}{16}} = 7\div\frac{3}{16} = 7\times\frac{16}{3} = \frac{7\times16}{3} = \frac{112}{3} = 37\frac{1}{3}$. Ans.

The line between 7 and $\frac{3}{16}$ means that the 7 is to be divided by $\frac{3}{16}$.

(*b*) $\frac{\frac{15}{32}}{\frac{5}{8}} = \frac{15}{32}\div\frac{5}{8} = \frac{15}{32}\times\frac{8}{5} = \frac{\overset{3}{\cancel{15}}\times\cancel{8}}{\underset{4}{\cancel{32}}\times\cancel{5}} = \frac{3}{4}$. Ans.

(*c*) $\frac{\frac{4+3}{2+6}}{5} = \frac{\frac{7}{8}}{5} = \frac{7}{8\times5} = \frac{7}{40}$. (See Art. **131.**) Ans.

(43) $\frac{7}{8}$ = value of the fraction, and 28 = the numerator. We find that 4 multiplied by 7 = 28, so multiplying 8, the denominator of the fraction, by 4, we have 32 for the required denominator, and $\frac{28}{32} = \frac{7}{8}$. Hence, 32 is the required denominator. Ans.

(44) (*a*) $\frac{7}{8}-\frac{7}{16}=?$ When the denominators of fractions are not alike, it is evident that the units are divided into unequal parts; therefore, before subtracting, reduce the fractions to fractions having a common denominator. Then, subtract the numerators and place the remainder over the common denominator.

$$\frac{7\times2}{8\times2}=\frac{14}{16}.\quad \frac{14}{16}-\frac{7}{16}=\frac{14-7}{16}=\frac{7}{16}.\quad \text{Ans.}$$

(*b*) $13-7\frac{7}{16}=?$ This problem may be solved in two ways.

First: $13=12\frac{16}{16}$, since $\frac{16}{16}=1$, and $12\frac{16}{16}=12+\frac{16}{16}=12+1=13$.

$$\begin{array}{r}12\frac{16}{16}\\ \underline{7\frac{7}{16}}\\ 5\frac{9}{16}\end{array}$$

We can now subtract the whole numbers separately, and the fractions separately, and obtain $12-7=5$, and $\frac{16}{16}-\frac{7}{16}=\frac{16-7}{16}=\frac{9}{16}$. $5+\frac{9}{16}=5\frac{9}{16}$. Ans.

Second: By reducing both numbers to improper fractions having a denominator of 16.

$$13=\frac{13}{1}=\frac{13\times16}{1\times16}=\frac{208}{16}.\quad 7\frac{7}{16}=\frac{(7\times16)+7}{16}=\frac{112+7}{16}=\frac{119}{16}.$$

Subtracting, we have $\frac{208}{16}-\frac{119}{16}=\frac{208-119}{16}=\frac{89}{16}$ and $\frac{89}{16}$,

$$=16\,)\,89\,(\,5\tfrac{9}{16}\qquad \begin{array}{r}\underline{80}\\ 9\end{array}$$

the same result that was obtained by the first method. Ans.

(*c*) $312\frac{9}{16}-229\frac{5}{32}=?$ We first reduce the fractions of the two mixed numbers to fractions having a common denominator. Doing this, we have $\frac{9}{16}=\frac{9\times2}{16\times2}=\frac{18}{32}$. We can now subtract the whole numbers and fractions separately, and have $312-229=83$, and $\frac{18}{32}-\frac{5}{32}=\frac{18-5}{32}=\frac{13}{32}$.

$$\begin{array}{r}312\frac{18}{32}\\ \underline{229\frac{5}{32}}\\ 83\frac{13}{32}\end{array}\qquad 83+\frac{13}{32}=83\frac{13}{32}.\quad \text{Ans.}$$

(45) The man evidently traveled $85\frac{5}{12}+78\frac{9}{15}+125\frac{17}{35}$ miles. Adding the fractions separately in this case,

$$\frac{5}{12}+\frac{9}{15}+\frac{17}{35}=\frac{5}{12}+\frac{3}{5}+\frac{17}{35}=\frac{175+252+204}{420}=\frac{631}{420}=1\frac{211}{420}.$$

Adding the whole numbers and the mixed number representing the sum of the fractions, the sum is $289\frac{211}{420}$ miles. Ans.

$$\begin{array}{r} 85 \\ 78 \\ 125 \\ 1\frac{211}{420} \\ \hline 289\frac{211}{420} \end{array}$$

To find the least common denominator, we have

$$\begin{array}{r|lll} 5 & 12, & 5, & 35 \\ \hline 7 & 12, & 1, & 7 \\ \hline & 12, & 1, & 1, \text{ or } 5\times 7\times 12 = 420. \end{array}$$

(46)

$$\begin{array}{lrl} & 573\frac{4}{5} & \text{tons.} \\ & 216\frac{5}{8} & \text{tons.} \\ \hline \textit{difference} & 357\frac{7}{40} & \text{tons. Ans.} \end{array} \qquad \begin{array}{l} \frac{4}{5}=\frac{32}{40} \\ \frac{5}{8}=\frac{25}{40} \\ \hline \frac{7}{40} = \textit{difference}. \end{array}$$

(47) Reducing $9\frac{1}{4}$ to an improper fraction, it becomes $\frac{37}{4}$. Multiplying $\frac{37}{4}$ by $\frac{3}{8}$, $\frac{37}{4}\times\frac{3}{8}=\frac{111}{32}=3\frac{15}{32}$ dollars. Ans.

(48) Referring to Arts. **114** and **116**, $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{11}$ of $\frac{19}{20}$ of 11 multiplied by $\frac{7}{8}$ of $\frac{5}{6}$ of 45

$$=\frac{\cancel{2}\times\cancel{3}\times 7\times 19\times\cancel{11}\times 7\times 5\times\overset{\overset{3}{\cancel{15}}}{\cancel{45}}}{\cancel{3}\times 4\times\cancel{11}\times\underset{4}{\cancel{20}}\times 1\times 8\times\underset{\cancel{3}}{\cancel{6}}\times 1}=\frac{7\times 19\times 7\times 5\times 3}{4\times 4\times 8}=\frac{13{,}965}{128}$$

$= 109\frac{13}{128}$. Ans.

(49) $\frac{3}{4}$ of $16=\frac{3}{\cancel{4}}\times\frac{\overset{4}{\cancel{16}}}{1}=12.\quad 12\div\frac{2}{3}=\frac{\overset{6}{\cancel{12}}}{1}\times\frac{3}{\cancel{2}}=18.$ Ans.

(50) $211\frac{1}{4}\times 1\frac{7}{8}=\frac{845}{4}\times\frac{15}{8}$, reducing the mixed numbers to improper fractions. $\frac{845}{4}\times\frac{15}{8}=\frac{12{,}675}{32}$ cents = amount paid for the lead. The number of pounds sold is evidently $\frac{12{,}675}{32}\div 2\frac{1}{2}=\frac{\overset{2{,}535}{\cancel{12{,}675}}}{\underset{16}{\cancel{32}}}\times\frac{\cancel{2}}{\cancel{5}}=\frac{2{,}535}{16}=158\frac{7}{16}$ pounds. The

amount remaining is $211\frac{1}{4} - 158\frac{7}{16} = \frac{845}{4} - \frac{2,535}{16} = \frac{3,380}{16} - \frac{2,535}{16} = \frac{845}{16} = 52\frac{13}{16}$ pounds. Ans.

	tenths.	hundredths.	
(51)	.0	8	= *Eight* **hundredths.**

tenths.	hundredths.	thousandths.	
.1	3	1	= *One hundred thirty-one* **thousandths.**

tenths.	hundredths.	thousandths.	ten-thousandths.	
.0	0	0	1	= *One* **ten-thousandth.**

tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	
.0	0	0	0	2	7	= *Twenty-seven* **millionths.**

tenths.	hundredths.	thousandths.	ten-thousandths.	
.0	1	0	8	= *One hundred eight* **ten-thousandths.**

	tenths.	hundredths.	thousandths.	ten-thousandths.	
93.	0	1	0	1	= Ninety-three, and *one hundred one* **ten-thousandths.**

In reading decimals, read the number just as you would if there were no ciphers before it. Then count from the decimal point towards the right, beginning with tenths, to as many places as there are figures, and the *name* of the last figure must be annexed to the previous reading of the figures to give the decimal reading. Thus, in the first example above, the simple reading of the figure is *eight*, and the name of its position in the decimal scale is **hundredths**, so that the decimal reading is *eight* **hundredths.** Similarly, the figures in the fourth example are ordinarily read *twenty-seven;* the name of the position of the figure 7 in the decimal scale is **millionths**, giving, therefore, the decimal reading as *twenty-seven* **millionths.**

If there should be a whole number before the decimal point, read it as you would read any whole number, and read the decimal as you would if the whole number were not there; or, read the whole number and then say, "and" so many hundredths, thousandths, or whatever it may be, as "ninety-three, and one hundred one ten-thousandths."

(52) See Art. **139.**

(53) See Art. **153.**

(54) See Art. **160.**

(55) A fraction is one or more of the equal parts of a unit, and is expressed by a numerator and a denominator, while a decimal fraction is a number of tenths, hundredths, thousandths, etc. of a unit, and is expressed by placing a period (.), called a decimal point, to the left of the figures of the number, and omitting the denominator.

(56) See Art. **165.**

(57) To reduce the fraction $\frac{1}{2}$ to a decimal, we annex one cipher to the numerator, which makes it 1.0. Dividing 1.0, the numerator, by 2, the denominator, gives a quotient of .5, the decimal point being placed before the *one* figure of the

quotient, or .5, since only one cipher was annexed to the numerator.

$$\frac{7}{8}\qquad \begin{array}{r} 8\,)\,\underline{7.000}\\ .875 \end{array}$$

$$\frac{5}{32}\qquad \begin{array}{l} 32\,)\,5.00000\,(\,.15625 \quad \text{Ans.}\\ \quad\ \underline{32}\\ \quad\ 180\\ \quad\ \underline{160}\\ \quad\ \ \ 200\\ \quad\ \ \ \underline{192}\\ \qquad\ \ \ 80\\ \qquad\ \ \ \underline{64}\\ \qquad\ \ \ 160\\ \qquad\ \ \ \underline{160} \end{array}$$

Since $.65 = \frac{65}{100}$, then, $\frac{65}{100}$ must equal .65. Or, when the denominator is 10, 100, 1,000, etc., point off as many places in the numerator as there are ciphers in the denominator. Doing so, $\frac{65}{100} = .65$. Ans.

$\frac{125}{1000} = .125$. Ans.

(58) (*a*) This example, written in the form of a fraction, means that the numerator $(32.5+.29+1.5)$ is to be divided by the denominator $(4.7+9)$. The operation is as follows:

$$\frac{32.5+.29+1.5}{4.7+9} = ?$$

$$\begin{array}{r} 32.5\\ +\ \ .29\\ +\ \ \underline{1.5} \end{array}$$

$$\begin{array}{l} 13.7\,)\,34.29000\,(\,2.5029 \quad \text{Ans.}\\ \qquad\ \underline{274}\\ \qquad\ \ 689\\ \qquad\ \ \underline{685}\\ \qquad\qquad 400\\ \qquad\qquad \underline{274}\\ \qquad\qquad 1260\\ \qquad\qquad \underline{1233}\\ \qquad\qquad\quad\ 27 \end{array}$$

$$\begin{array}{r} 4.7\\ +\ \underline{9.0}\\ 13.7 \end{array}$$

Since there are 5 decimal places in the dividend, and 1 in the divisor, there are 5 − 1 or 4 places to be pointed off in the quotient. The fifth figure of the decimal is evidently less than 5.

(*b*) Here again the problem is to divide the numerator, which is $(1.283\times\overline{8+5})$, by the denominator, which is 2.63. The operation is as follows:

$$\frac{1.283\times\overline{8+5}}{2.63} = ? \quad \overline{8+5} = 13.$$

$$\begin{array}{r}
1.283 \\
\times\quad 13 \\
\hline
3849 \\
1283 \\
\hline
\end{array}$$

$$\begin{array}{rl}
2.63\,) & 16.679000\,(\,6.3418 \quad \text{Ans.} \\
 & \underline{1578} \\
 & 899 \\
 & \underline{789} \\
 & 1100 \\
 & \underline{1052} \\
 & 480 \\
 & \underline{263} \\
 & 2170 \\
 & \underline{2104} \\
 & 66
\end{array}$$

(*c*) $\dfrac{\overline{589+27}\times\overline{163-8}}{25+39} = ?$

$$\begin{array}{r}
589 \\
+\ \ 27 \\
\hline
616
\end{array}
\qquad\qquad
\begin{array}{r}
163 \\
-\quad 8 \\
\hline
155 \\
\times 616 \\
\hline
930 \\
155 \\
930 \\
\hline
95480
\end{array}$$

$$\begin{array}{r}25\\+39\\\hline 64\end{array}$$

$$\begin{array}{rcccccccc l}
64) & 9 & 5 & 4 & 8 & 0. & 0 & 0 & 0 & (1491.875\\
 & 6 & 4 & & & & & & & \\ \cline{2-3}
 & 3 & 1 & 4 & & & & & & \\
 & 2 & 5 & 6 & & & & & & \\ \cline{2-4}
 & & 5 & 8 & 8 & & & & & \\
 & & 5 & 7 & 6 & & & & & \\ \cline{3-5}
 & & & 1 & 2 & 0 & & & & \\
 & & & & 6 & 4 & & & & \\ \cline{4-6}
 & & & & 5 & 6 & 0 & & & \\
 & & & & 5 & 1 & 2 & & & \\ \cline{5-7}
 & & & & & 4 & 8 & 0 & & \\
 & & & & & 4 & 4 & 8 & & \\ \cline{6-8}
 & & & & & & 3 & 2 & 0 & \\
 & & & & & & 3 & 2 & 0 & \\ \cline{7-9}
\end{array}$$

There are three decimal places in the quotient, since three ciphers were annexed to the dividend.

$$(d)\quad \frac{\overline{40.6+7.1}\times(3.029-1.874)}{6.27+8.53-8.01}=?$$

$$\begin{array}{r}40.6\\+\ \ 7.1\\\hline 47.7\end{array}\qquad\qquad \begin{array}{r}3.029\\-1.874\\\hline 1.155\\\times\ \ 47.7\\\hline 8085\\8085\\4620\end{array}$$

$$\begin{array}{r}6.27\\+\ \ 8.53\\\hline 14.80\\-\ \ 8.01\\\hline 6.79\end{array}$$

$$\begin{array}{rcccccccc l}
6.79) & 5 & 5. & 0 & 9 & 3 & 5 & 0 & 0 & (8.1139\ \ \text{Ans}\\
 & 5 & 4 & 3 & 2 & & & & & \\ \cline{2-5}
 & & & 7 & 7 & 3 & & & & \\
 & & & 6 & 7 & 9 & & & & \\ \cline{4-6}
 & & & & 9 & 4 & 5 & & & \\
 & & & & 6 & 7 & 9 & & & \\ \cline{5-7}
 & & & & 2 & 6 & 6 & 0 & & \\
 & & & & 2 & 0 & 3 & 7 & & \\ \cline{5-8}
 & & & & & 6 & 2 & 3 & 0 & \\
 & & & & & 6 & 1 & 1 & 1 & \\ \cline{6-9}
 & & & & & & 1 & 1 & 9 & \\
\end{array}$$

6 decimal places in the dividend — 2 decimal places in the divisor = 4 decimal places to be pointed off in the quotient.

(59) $.875 = \frac{875}{1000} = \frac{175}{200} = \frac{7}{8}$ of a foot.

1 foot = 12 inches.

$$\frac{7}{8} \text{ of 1 foot} = \frac{7}{\underset{2}{\cancel{8}}} \times \frac{\overset{3}{\cancel{12}}}{1} = \frac{21}{2} = 10\tfrac{1}{2} \text{ inches.} \quad \text{Ans.}$$

(60) 12 inches = 1 foot.

$$\frac{3}{16} \text{ of an inch} = \frac{3}{16} \div 12 = \frac{\cancel{3}}{16} \times \frac{1}{\underset{4}{\cancel{12}}} = \frac{1}{64} \text{ of a foot.}$$

```
 1
 --
 64 ) 1.0 0 0 0 0 0 ( .0 1 5 6 2 5   Ans.
      6 4
      -----
      3 6 0
      3 2 0
      -------
        4 0 0
        3 8 4
        -------
          1 6 0
          1 2 8
          -------
            3 2 0
            3 2 0
            -----
```

Point off 6 decimal places in the quotient, since we annexed six ciphers to the dividend, the divisor containing no decimal places; hence, 6 − 0 = 6 places to be pointed off.

(61) If 1 cubic inch of water weighs .03617 of a pound, the weight of 1,500 cubic inches will be .03617 × 1,500 = 54.255 lb.

```
   .0 3 6 1 7 lb.
       1 5 0 0
   -------------
   1 8 0 8 5 0 0
   3 6 1 7
   -------------
   5 4.2 5 5 0 0 lb.   Ans.
```

(62) 72.6 feet of fencing at \$.50 a foot would cost 7 2.6 × .50, or \$36.30.

```
      .5 0
   ----------
   $ 3 6.3 0 0
```

If, by selling a carload of coal at a profit of \$1.65 per ton, I make \$36.30, then there must be as many tons of coal in the car as 1.65 is contained times in 36.30, or 22 tons.

$$\begin{array}{r l}
1.65\,)\,36.30\,(\,22 \text{ tons.} & \text{Ans.} \\
\underline{330} & \\
330 & \\
\underline{330} &
\end{array}$$

(63)

$$\begin{array}{r}
231\,)\,17892.00000\,(\,77.45454 \\
\underline{1617} \\
1722 \\
\underline{1617} \\
1050 \\
\underline{924} \\
1260 \\
\underline{1155} \\
1050 \\
\underline{924} \\
1260 \\
\underline{1155} \\
1050
\end{array}$$

or 77.4545 to four decimal places. Ans.

(64)

$$\frac{\overset{37.13}{\cancel{74.26}} \times \overset{\cancel{2}}{\cancel{24}} \times \overset{.0952}{\cancel{3.1416}} \times 19 \times 19 \times 350}{\underset{1,000}{\cancel{33,000}} \times \cancel{12} \times \underset{2}{\cancel{4}}}$$

$$= \frac{37.13 \times .0952 \times 19 \times 19 \times 350}{1,000} = \frac{446,618.947600}{1,000}$$

$= 446.619$ to three decimal places. Ans.

$$\begin{array}{r}
37.13 \\
\underline{.0952} \\
7426 \\
18565 \\
\underline{33417} \\
3.534776
\end{array}
\qquad
\begin{array}{r}
19 \\
\underline{19} \\
171 \\
\underline{19} \\
361
\end{array}
\qquad
\begin{array}{r}
361 \\
\underline{350} \\
18050 \\
\underline{1083} \\
126350
\end{array}
\qquad
\begin{array}{r}
3.534776 \\
\underline{126350} \\
176738800 \\
10604328 \\
21208656 \\
7069552 \\
\underline{3534776} \\
446618.947600
\end{array}$$

(65) See Art. **174.** Applying rule in Art. **175,**

(*a*) $.7928 \times \frac{64}{64} = \frac{50.7392}{64} = \frac{51}{64}$. Ans.

(*b*) $.1416 \times \frac{32}{32} = \frac{4.5312}{32} = \frac{5}{32}$. Ans.

(*c*) $.47915 \times \frac{16}{16} = \frac{7.6664}{16} = \frac{8}{16} = \frac{1}{2}$. Ans.

(66) In subtraction of decimals, place the decimal points directly under each other, and proceed as in the subtraction of whole numbers, placing the decimal point in the remainder directly under the decimal points above.

(*a*)
$$\begin{array}{r} 709.6300 \\ .8514 \\ \hline 708.7786 \end{array} \text{ Ans.}$$

In the above example we proceed as follows: We cannot subtract 4 ten-thousandths from 0 ten-thousandths, and as there are no thousandths, we take 1 hundredth from the 3 hundredths. 1 hundredth = 10 thousandths = 100 ten-thousandths. 4 ten-thousandths from 100 ten-thousandths leaves 96 ten-thousandths. 96 ten-thousandths = 9 thousandths + 6 ten-thousandths. Write the 6 ten-thousandths in the ten-thousandths place in the remainder. The next figure in the subtrahend is 1 thousandth. This must be subtracted from the 9 thousandths, which is a part of the 1 hundredth taken previously from the 3 hundredths. Subtracting, we have 1 thousandth from 9 thousandths leaves 8 thousandths, the 8 being written in its place in the remainder. Next we have to subtract 5 hundredths from 2 hundredths (1 hundredth having been taken from the 3 hundredths makes it but 2 hundredths now). Since we cannot do this, we take 1 tenth from 6 tenths. 1 tenth (= 10 hundredths) + 2 hundredths = 12 hundredths. 5 hundredths from 12 hundredths leaves 7 hundredths. Write the 7 in the hundredths place in the remainder. Next we have to subtract 8 tenths from 5 tenths (5 tenths now, because 1 tenth was taken from the 6 tenths). Since this cannot be done, we take 1 unit from the 9 units. 1 unit = 10 tenths; 10 tenths + 5 tenths = 15 tenths, and 8 tenths from 15 tenths leaves 7 tenths. Write the 7 in the tenths place in the remainder. In the minuend we now have 708 units (1 unit having been taken away) and 0 units in the subtrahend. 0 units from 708 units leaves 708 units; hence, we write 708 in the remainder.

(*b*) $$\begin{array}{r}81.963\\1.700\\\hline 80.263\end{array}\text{ Ans.}$$

(*c*) $$\begin{array}{r}18.00\\.18\\\hline 17.82\end{array}\text{ Ans.}$$

(*d*) $$\begin{array}{r}1.000\\.001\\\hline .999\end{array}\text{ Ans.}$$

(*e*) $872.1-(.8721+.008)=?$ In this problem we are to subtract $(.8721+.008)$ from 872.1. First perform the operation as indicated by the sign between the decimals enclosed by the parenthesis.

$$\begin{array}{r}.8721\\.0080\\\hline .8801\end{array}\textit{ sum.}$$

Subtracting the sum (obtained by adding the decimals enclosed within the parenthesis) from the number 872.1 (as required by the minus sign before the parenthesis), we obtain the required remainder.

$$\begin{array}{r}872.1000\\.8801\\\hline 871.2199\end{array}\text{ Ans.}$$

(*f*) $(5.028+.0073)-(6.704-2.38)=?$ First perform the operations as indicated by the signs between the numbers enclosed by the parentheses. The first parenthesis shows that 5.028 and .0073 are to be added. This gives 5.0353 as their sum.

$$\begin{array}{r}5.0280\\.0073\\\hline 5.0353\end{array}\textit{ sum.}$$

The second parenthesis shows that 2.38 is to be subtracted from 6.704. The difference is found to be 4.324.

$$\begin{array}{r}6.704\\2.380\\\hline 4.324\end{array}\textit{ difference.}$$

The sign between the parentheses indicates that the quantities obtained by performing the above operations are to be subtracted, namely, that 4.324 is to be subtracted from 5.0353. Performing this operation, we obtain .7113 as the final result.

$$\begin{array}{r}5.0353\\4.3240\\\hline .7113\end{array}\text{ Ans.}$$

(67) In subtracting a decimal from a fraction, or in subtracting a fraction from a decimal, either reduce the

fraction to a decimal before subtracting or reduce the decimal to a fraction and then subtract.

(*a*) $\frac{7}{8} - .807 = ?$ $\frac{7}{8}$ reduced to a decimal becomes $\frac{7}{8}$ $\begin{array}{r} 8\,\underline{)\;7.0\,0\,0} \\ .8\,7\,5 \end{array}$

$$\begin{array}{r} .8\,7\,5 \\ \underline{.8\,0\,7} \\ .0\,6\,8 \end{array} \text{ Ans.}$$

Subtracting .807 from .875, the remainder is .068, as shown.

(*b*) $.875 - \frac{3}{8} = ?$ Reducing .875 to a fraction, we have $.875 = \frac{875}{1000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$; hence, $\frac{7}{8} - \frac{3}{8} = \frac{7-3}{8} = \frac{4}{8} = \frac{1}{2}$, or, .5. Ans.

Or, by reducing $\frac{3}{8}$ to a decimal, $\frac{3}{8}$ $\begin{array}{r} 8\,\underline{)\;3.0\,0\,0} \\ .3\,7\,5 \end{array}$ and then subtracting, we obtain $.875 - .375 = .5 = \frac{5}{10} = \frac{1}{2}$, the same answer as above.

$$\begin{array}{r} .8\,7\,7 \\ \underline{.3\,7\,5} \\ .5\,0\,0 \end{array} \text{ Ans.}$$

(*c*) $(\frac{5}{32} + .435) - (\frac{21}{100} - .07) = ?$ We first perform the operations as indicated by the signs between the numbers enclosed by the parentheses. Reduce $\frac{5}{32}$ to a decimal and we obtain $\frac{5}{32} = .15625$ (see example 60).

Adding .15625 and .435,

$$\begin{array}{r} .1\,5\,6\,2\,5 \\ \underline{.4\,3\,5} \\ \textit{sum}\ .5\,9\,1\,2\,5 \end{array} \qquad \frac{21}{100} = .21;\ \text{subtracting,} \quad \begin{array}{r} .2\,1 \\ \underline{.0\,7} \\ \textit{difference}\ .1\,4 \end{array}$$

We are now prepared to perform the operation indicated by the minus sign between the parentheses, which is,

$$\begin{array}{r} .5\,9\,1\,2\,5 \\ \underline{.1\,4} \\ .4\,5\,1\,2\,5 \end{array} \text{ Ans.}$$

(*d*) This problem means that 33 millionths and 17 thousandths are to be added. Also, that 53 hundredths and 274 thousandths are to be added, and the smaller of these sums is to be subtracted from the larger sum. Thus $(.53 + .274) - (.000033 + .017) = ?$

tenths.
hundredths.
thousandths.
ten-thousandths.
hundred-thousandths
mill.onths.
.0 0 0 0 3 3
.0 1 7
.0 1 7 0 3 3 *sum.*

tenths.
hundredths.
thousandths.
.5 3
.2 7 4
.8 0 4 *sum.*

.8 0 4 *larger sum.*
.0 1 7 0 3 3 *smaller sum.*
difference .7 8 6 9 6 7 Ans.

(68) In addition of decimals the decimal points must be placed directly under each other, so that tenths will come under tenths, hundredths under hundredths, thousandths under thousandths, etc. The addition is then performed as in whole numbers, the decimal point of the sum being placed directly under the decimal points above.

.1 2 5
.7
.0 8 9
.4 0 0 5
.9
.0 0 0 0 2 7
2.2 1 4 5 2 7 Ans.

(69)
9 2 7.4 1 6
8.2 7 4
3 7 2.6
6 2.0 7 9 3 8
1 3 7 0.3 6 9 3 8 Ans.

(70)
tenths.
hundredths.
thousandths.
ten-thousandths.
hundred-thousandths.
millionths.
.0 1 7
.2
.0 0 0 0 4 7
.2 1 7 0 4 7 Ans.

(71) (*a*) There are 3 decimal places in the multiplicand and 3 in the multiplier; hence, there are 3 + 3 or 6 decimal places in the product. Since the product contains but four figures, we prefix two ciphers in order to obtain the necessary six decimal places.

.1 0 7
.0 1 3
3 2 1
1 0 7
.0 0 1 3 9 1 Ans.

(*b*)

$$\begin{array}{r}
203\\
2.03\\ \hline
609\\
4060\\ \hline
412.09\\
.203\\ \hline
123627\\
824180\\ \hline
83.65427
\end{array}\quad \text{Ans.}$$

There are 2 decimal places in the multiplier and none in the multiplicand; hence, there are 2 + 0 or 2 decimal places in the first product.

Since there are 2 decimal places in the multiplicand and 3 decimal places in the multiplier, there are 3 + 2 or 5 decimal places in the second product.

(*c*) First perform the operations indicated by the signs between the numbers enclosed by the parenthesis, and then perform whatever may be required by the sign before the parenthesis.

Multiply together the numbers 2.7 and 31.85.

$$\begin{array}{r}
31.85\\
2.7\\ \hline
22295\\
6370\\ \hline
85.995
\end{array}$$

The parenthesis shows that .316 is to be taken from 3.16.

$$\begin{array}{r}
3.160\\
.316\\ \hline
2.844
\end{array}$$

The product obtained by the first operation is now multiplied by the remainder obtained by performing the operation indicated by the signs within the parenthesis.

$$\begin{array}{r}
85.995\\
2.844\\ \hline
343980\\
343980\\
687960\\
171990\\ \hline
244.569780
\end{array}\quad \text{Ans.}$$

(*d*) $(107.8 + 6.541 - 31.96) \times 1.742 = ?$

$$\begin{array}{r}
107.8\\
+\quad 6.541\\ \hline
114.341\\
-\quad 31.96\\ \hline
82.381
\end{array}
\qquad\qquad
\begin{array}{r}
82.381\\
\times\quad 1.742\\ \hline
164762\\
329524\\
576667\\
82381\\ \hline
143.507702
\end{array}\quad \text{Ans.}$$

(72) (*a*) $(\frac{7}{16}-.13)\times\overline{.625+\frac{5}{8}}=?$

First perform the operation indicated by the parenthesis.

$$\frac{7}{16}=\frac{7}{16}\,)\,7.0000\,(.4375$$

$$\begin{array}{r} 64 \\ \hline 60 \\ 48 \\ \hline 120 \\ 112 \\ \hline 80 \\ 80 \\ \hline \end{array}$$

We point off four decimal places, since we annex four ciphers.

$$\begin{array}{r} .4375 \\ .13 \\ \hline \end{array}$$

Subtracting, we obtain $.3075$

The vinculum has the same meaning as the parenthesis; hence, we perform the operation indicated by it. We point off three decimal places, since three ciphers were annexed to the 5.

$$\frac{5}{8}=\frac{5}{8}\,)\,\underline{5.000}\qquad .625$$

Adding the terms included by the vinculum, we obtain

$$\begin{array}{r} .625 \\ .625 \\ \hline 1.250 \end{array}$$

The final operation is to perform the work indicated by the sign between the parenthesis and the vinculum, thus,

$$\begin{array}{r} .3075 \\ 1.25 \\ \hline 15375 \\ 6150 \\ 3075 \\ \hline .384375 \end{array}\quad \text{Ans.}$$

(*b*) $(\frac{19}{32}\times.21)-(.02\times\frac{3}{16})=?$

$.21=\frac{21}{100}$. $\frac{19}{32}\times\frac{21}{100}=\frac{399}{3200}$. $.02=\frac{2}{100}$. $\frac{2}{100}\times\frac{3}{16}=\frac{6}{1600}$

$=\frac{3}{800}$. $\frac{3}{800}=\frac{3\times4}{800\times4}=\frac{12}{3200}$. $\frac{399}{3200}-\frac{12}{3200}$

$$=\frac{399-12}{3,200}=\frac{387}{3200}.$$

Reducing $\frac{387}{3200}$ to a decimal, we obtain

$\frac{387}{3,200}$) 3 8 7.0 0 0 0 0 0 0 (.1 2 0 9 3 7 5 Ans.
3 2 0 0
6 7 0 0
6 4 0 0
3 0 0 0 0
2 8 8 0 0
1 2 0 0 0
9 6 0 0
2 4 0 0 0
2 2 4 0 0
1 6 0 0 0
1 6 0 0 0

Point off seven decimal places, since seven ciphers were annexed to the dividend.

(*c*) $(\frac{13}{4}+.013-2.17)\times\overline{13\frac{1}{4}-7\frac{5}{16}}=?$

$\frac{13}{4}=\frac{13}{4}$) 1 3.0 0
3.2 5

Point off two decimal places, since two ciphers were annexed to the dividend.

3.2 5
$+$ .0 1 3
3.2 6 3
$-$ 2.1 7
1.0 9 3

$\frac{5}{16}$ reduced to a decimal is .3125, since

$\frac{5}{16}$) 5.0 0 0 0 (.3 1 2 5
4 8
2 0
1 6
4 0
3 2
8 0
8 0

Point off four decimal places, since four ciphers were annexed to the dividend.

Then, $7\frac{5}{16}=7.3125$, and $13\frac{1}{4}=13.25$, since $\frac{1}{4}=\frac{1}{4}$) 1.0 0
.2 5

1 3.2 5
$-$ 7.3 1 2 5
5.9 3 7 5

5.9 3 7 5
$\times$ 1.0 9 3
1 7 8 1 2 5
5 3 4 3 7 5
5 9 3 7 5 0
6.4 8 9 6 8 7 5 Ans.

(73) (*a*) $.875 \div \frac{1}{2} = .875 \div .5$ (since $\frac{1}{2} = .5$) $= 1.75$. Ans.

Another way of solving this is to reduce .875 to its equivalent common fraction and then divide.

$.875 = \frac{7}{8}$, since $.875 = \frac{875}{1000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$; then,

$$\frac{7}{8} \div \frac{1}{2} = \frac{7}{\underset{4}{\cancel{8}}} \times \frac{\cancel{2}}{1} = \frac{7}{4} = 1\frac{3}{4}.$$

Since $\frac{3}{4} = \frac{3}{4}$) 3.0 0 (.7 5, $1\frac{3}{4} = 1.75$, the same answer as above.

$$\begin{array}{r} 4\,)\,3.0\,0\,(.7\,5 \\ \underline{2\,8} \\ 2\,0 \\ \underline{2\,0} \end{array}$$

(*b*) $\frac{7}{8} \div .5 = \frac{7}{8} \div \frac{1}{2}$ (since $.5 = \frac{1}{2}$) $= \frac{7}{\underset{4}{\cancel{8}}} \times \frac{2}{1} = \frac{7}{4} = 1\frac{3}{4}$, or 1.75. Ans.

This can also be solved by reducing $\frac{7}{8}$ to its equivalent decimal and dividing by .5; $\frac{7}{8} = .875$; $.875 \div .5 = 1.75$. Since there are 3 decimal places in the dividend and 1 in the divisor, there are 3 − 1 or 2 decimal places in the quotient.

(*c*) $\dfrac{.375 \times \frac{1}{4}}{\frac{5}{16} - .125} = ?$ We shall solve this problem by first reducing the decimals to their equivalent common fractions.

$.375 = \frac{375}{1000} = \frac{75}{200} = \frac{15}{40} = \frac{3}{8}$. $\frac{3}{8} \times \frac{1}{4} = \frac{3}{32}$, or the value of the numerator of the fraction.

$.125 = \frac{125}{1000} = \frac{25}{200} = \frac{1}{8}$. Reducing $\frac{1}{8}$ to 16ths, we have $\frac{1 \times 2}{8 \times 2} = \frac{2}{16}$. Then, $\frac{5}{16} - \frac{2}{16} = \frac{3}{16}$, or the value of the denominator of the fraction. The problem is now reduced to $\dfrac{\frac{3}{32}}{\frac{3}{16}} = ?$ $\dfrac{\frac{3}{32}}{\frac{3}{16}} = \frac{3}{32} \div \frac{3}{16} = \frac{\cancel{3}}{\underset{2}{\cancel{32}}} \times \frac{\cancel{16}}{\cancel{3}} = \frac{1}{2}$ or .5. Ans.

(74) $\dfrac{\dfrac{1.25 \times 20 \times 3}{87 + (11 \times 8)}}{459 + 32} = ?$ In this problem $1.25 \times 20 \times 3$ constitutes the numerator of the complex fraction.

Multiplying the factors of the numerator together, we find their product to be 75.

$$\begin{array}{r} 1.2\,5 \\ \times\ \underline{2\,0} \\ 2\,5.0\,0 \\ \times\ \underline{3} \\ 7\,5 \end{array}$$

The fraction $\frac{87+(11\times 8)}{459+32}$ constitutes the denominator of the complex fraction. The value of the numerator of this fraction equals $87+88 = 175$.

The numerator is combined as though it were written $87+(11\times 8)$, and its result is

$$\begin{array}{r} 11 \\ \times\ \underline{8} \\ 88 \\ +\ \underline{87} \\ 175 \end{array}$$

The value of the denominator of this fraction is equal to $459+32 = 491$. The problem then becomes

$$\frac{75}{\frac{175}{491}} = \tfrac{75}{1} \div \tfrac{175}{491} = \tfrac{75}{1} \times \tfrac{491}{175} = \frac{\overset{3}{\cancel{75}} \times 491}{\underset{7}{\cancel{175}}} = \tfrac{1473}{7} = 210\tfrac{3}{7}.$$

Ans.

(75) 1 plus .001 = 1.001. .01 plus .000001 = .010001. And 1.001 − .010001 = .990999. Ans.

ARITHMETIC.

(QUESTIONS 1-92. SEC. 2.)

(1) A certain per cent. of a number means so many hundredths of that number.

25% of 8,428 lb. means 25 hundredths of 8,428 lb. Hence, 25% of 8,428 lb. = .25×8,428 lb. = 2,107 lb. Ans.

(2) Here \$100 is the base and 1% = .01 is the rate. Then, .01×\$100 = \$1. Ans.

(3) ½% means one-half of 1 per cent. Since 1% is .01, ½% is .005, for 2).0 1 0. And .005×\$35,000 = \$175. Ans.
.0 0 5

(4) Here 50 is the base, 2 is the percentage, and it is required to find the rate. Applying rule, Art. **12,**

rate = percentage ÷ base;
rate = 2 ÷ 50 = .04, or 4%. Ans.

(5) By Art. **12,** rate = percentage ÷ base.*
As percentage = 10 and base = 10, we have

rate = 10 ÷ 10 = 1 = 100%.

Hence, 10 is 100% of 10. Ans.

---

* Remember that an expression of this form means that the first term is to be *divided by* the second term. Thus, as above, it means percentage *divided by* base.

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(6) (*a*) Rate = percentage ÷ base. Art. **12.**

As percentage = **$176.54** and base = **$2,522**, we **have**

rate = 176.54 ÷ 2,522 = .07 = 7%. Ans.

$$\begin{array}{r|l} 2522 & 176.54 \\ \hline & .07 \end{array}$$

(*b*) **Base** = percentage ÷ rate. Art. **11.**

As percentage = 16.96 and rate = 8% = .08, **we have**

base = 16.96 ÷ .08 = 212. Ans.

$$\begin{array}{r|l} .08 & 16.96 \\ \hline & 212 \end{array}$$

(*c*) Amount is the sum of the base and percentage; hence, the percentage = amount minus the base.

Amount = 216.7025 and base = 213.5; hence, percentage = 216.7025 − 213.5 = 3.2025.

Rate = percentage ÷ base. Art. **12.**

Therefore, rate = 3.2025 ÷ 213.5 = .015 = $1\frac{1}{2}$%. Ans.

$$\begin{array}{l} 213.5)\,3.2025\,(.015 = 1\tfrac{1}{2}\%. \\ \quad\quad\quad\underline{2135} \\ \quad\quad\quad10675 \\ \quad\quad\quad\underline{10675} \end{array}$$

(*d*) The difference is the remainder found by subtracting the percentage from the base; hence, base minus the difference = the percentage. Base = 207 and difference = 201.825; hence, percentage = 207 − 201.825 = 5.175.

Rate = percentage ÷ base. Art. **12.**

Therefore, rate = 5.175 ÷ 207 = .025 = .02$\frac{1}{2}$ = $2\frac{1}{2}$%. Ans.

$$\begin{array}{l} 207)\,5.175\,(.025 = 2\tfrac{1}{2}\%. \\ \quad\quad\quad\underline{414} \\ \quad\quad\quad1035 \\ \quad\quad\quad\underline{1035} \end{array}$$

(7) In this problem $5,500 is the amount, since it equals what he paid for the farm plus what he gained; 15% is the rate, and the cost (to be found) is the base. **Applying rule, Art. 16,**

base = amount ÷ (1 + rate); hence,
base = \$5,500 ÷ (1 + .15) = \$4,782.61. Ans.

```
1.1 5 ) 5 5 0 0.0 0 0 0 ( 4 7 8 2.6 1—
        4 6 0
        -----
          9 0 0
          8 0 5
          -----
            9 5 0
            9 2 0
            -----
              3 0 0
              2 3 0
              -----
                7 0 0
                6 9 0
                -----
                  1 0 0
                  1 1 5
                  -----
```

The example can also be solved as follows: 100% = cost; if he gained 15%, then 100 + 15 = 115% = \$5,500, the selling price.

If 115% = \$5,500, 1% = $\frac{1}{115}$ of \$5,500 = \$47.8261, and 100%, or the cost = 100 × \$47.8261 = \$4,782.61. Ans.

```
(8)  2 4 % of $950 = 950×.24  = $2 2 8.0 0
     1 2½% of $950 = 950×.125 =   1 1 8.7 5
     1 7 % of $950 = 950×.17  =   1 6 1.5 0
     ------                     -----------
     5 3½% of $950            = $5 0 8.2 5
```

The total amount of his yearly expenses, then, is \$508.25; hence, his savings are \$950 − \$508.25 = \$441.75. Ans.

Or, as above, 24% + 12½% + 17% = 53½%, the total percentage of expenditures; hence, 100% − 53½% = 46½% = per cent. saved. And \$950 × .465 = \$441.75 = his yearly savings. Ans.

(9) The percentage is 961.38, and the rate is .37½. By Art. **11**,

base = percentage ÷ rate
= 961.38 ÷ .375 = 2,563.68, the number. Ans.

```
.375)961.38000(2563.68
     750
     2113
     1875
      2388
      2250
       1380
       1125
        2550
        2250
         3000
         3000
```

Another method of solving is the following:

If $37\frac{1}{2}\%$ of a number is 961.38, then $.37\frac{1}{2}$ times the number = 961.38, and the number = $961.38 \div .37\frac{1}{2}$, which, as above = 2,563.68. Ans.

(10) Here $1,125 is 30% of some number; hence, $1,125 = the percentage, 30% = the rate, and the required number is the base. Applying rule, Art. **11**,

base = percentage ÷ rate = $1,125 ÷ .30 = $3,750.

Since $3,750 is $\frac{3}{4}$ of the property, one of the fourths is $\frac{1}{3}$ of $3,750 = $1,250, and 4 fourths, or the entire property, is 4 × $1,250 = $5,000. Ans.

(11) Here $4,810 is the difference and 35% the rate. By Art. **17**,

base = difference ÷ (1 − rate)

= $4,810 ÷ (1 − .35) = $4,810 ÷ .65 = $7,400. Ans.

```
.65)4810.00(7400      1.00
    455                .35
     260               .65
     260
```

Solution can also be effected as follows: 100% = the sum diminished by 35%, then (1 − .35) = .65, which is $4,810.

If 65% = $4,810, 1% = $\frac{1}{65}$ of $4,810 = $74, and 100% = 100 × $74 = $7,400. Ans.

(12) In this example the sales on Monday amounted to $197.55, which was $12\frac{1}{2}\%$ of the sales for the entire week; i.e., we have given the percentage, $197.55, and the rate, $12\frac{1}{2}\%$,

and the required number (or the amount of sales for the week) equals the base. By Art. **11**,

$$\text{base} = \text{percentage} \div \text{rate} = \$197.55 \div .125;$$

or,

$$\begin{array}{l}
.125)\,197.5500\,(1580.4 \\
\underline{125} \\
725 \\
\underline{625} \\
1005 \\
\underline{1000} \\
500 \\
\underline{500}
\end{array}$$

Therefore, base = $1,580.40, which also equals the sales for the week. Ans.

(13) 16.5 miles = 12½% of the entire length of the road. We wish to find the *entire* length.

16.5 miles is the percentage, 12½% is the rate, and the entire length will be the base. By Art. **11**,

$$\text{base} = \text{percentage} \div \text{rate} = 16.5 \div .12\tfrac{1}{2}.$$

$$\begin{array}{l}
.125)\,16.500\,(132 \text{ miles. Ans.} \\
\underline{125} \\
400 \\
\underline{375} \\
250 \\
\underline{250}
\end{array}$$

(14) Here we have given the difference, or $35, and the rate, or 60%, to find the base. We use the rule in Art. **17**,

$$\begin{aligned}
\text{base} &= \text{difference} \div (1 - \text{rate}) \\
&= \$35 \div (1 - .60) = \$35 \div .40 = \$87.50. \text{ Ans.}
\end{aligned}$$

$$\begin{array}{l}
.40)\,35.000\,(87.5 \\
\underline{320} \\
300 \\
\underline{280} \\
200 \\
\underline{200}
\end{array}$$

Or, 100% = whole debt; 100% − 60% = 40% = \$35.

If 40% = \$35, then 1% = $\frac{1}{40}$ of \$35 = $\frac{\$35}{40}$, and 100% = $\frac{\$35}{40} \times 100$ = \$87.50. Ans.

(15) 28 rd. 4 yd. 2 ft. 10 in. to inches.

$$\begin{array}{rrl} \times & 5\frac{1}{2} & \\ \hline & 154 & \\ + & 4 & \\ \hline & 158 & \text{yards.} \\ \times & 3 & \\ \hline & 474 & \\ + & 2 & \\ \hline & 476 & \text{feet.} \\ \times & 12 & \\ \hline & 5712 & \\ + & 10 & \\ \hline & 5722 & \text{inches.} \end{array}$$

Since there are 5½ yards in 1 rod, in 28 rods there are 28 × 5½, or 154 yards; 154 yards + 4 yards = 158 yards. There are 3 feet in 1 yard; therefore, in 158 yards there are 3 × 158, or 474 feet; 474 feet + 2 feet = 476 feet. There are 12 inches in 1 foot, and in 476 feet there are 12 × 476, or 5,712 inches; 5,712 inches + 10 inches = 5,722 inches. Ans.

(16)

$$\begin{array}{r@{\,)\,}l} 12 & \underline{5722} \text{ inches.} \\ 3 & \underline{476} + 10 \text{ inches.} \\ 5\frac{1}{2} & \underline{158} + 2 \text{ feet.} \\ \multicolumn{2}{r}{28 + 4 \text{ yards.}} \end{array}$$

28 rd. 4 yd. 2 ft. 10 in. Ans.

Explanation.—There are 12 inches in 1 foot; hence, in 5,722 inches there are as many feet as 12 is contained times in 5,722 inches, or 476 feet and 10 inches remaining. Write these 10 inches as a remainder. There are 3 feet in 1 yard; hence, in 476 feet there are as many yards as 3 is contained times in 476 feet, or 158 yards and 2 feet remaining. There are 5½ yards in 1 rod; hence, in 158 yards there are 28 rods and 4 yards remaining. Then, in 5,722 inches, there are 28 rd. 4 yd. 2 ft. 10 in.

(17)

$$\begin{array}{rrl} & 5 & \text{weeks } 3.5 \text{ days.} \\ \times & 7 & \\ \hline & 35 & \text{days in 5 weeks.} \\ + & 3.5 & \\ \hline & 38.5 & \text{days.} \end{array}$$

Then, we find how many seconds there are in 38.5 days.

$$\begin{array}{rl}
38.5 & \text{days.} \\
\times\quad 24 & \text{hours in 1 day.} \\
\hline
1540 & \\
770 & \\
\hline
924.0 & \text{hours in 38.5 days.} \\
\times\quad 60 & \text{minutes in 1 hour.} \\
\hline
55440 & \text{minutes in 38.5 days.} \\
\times\quad 60 & \text{seconds in 1 minute.} \\
\hline
3326400 & \text{seconds in 38.5 days.}\quad \text{Ans.}
\end{array}$$

(18) Since there are 24 gr. in 1 pwt., in 13,750 gr. there are as many pennyweights as 24 is contained times in 13,750, or 572 pwt. and 22 gr. remaining. Since there are 20 pwt. in 1 oz., in 572 pwt. there are as many ounces as 20 is contained times in 572, or 28 oz. and 12 pwt. remaining. Since there are 12 oz. in 1 lb. (Troy), in 28 oz. there are as many pounds as 12 is contained times in 28, or 2 lb. and 4 oz. remaining. We now have the pounds and ounces required by the problem; therefore, in 13,750 gr. there are 2 lb. 4 oz. 12 pwt. 22 gr.

$$\begin{array}{rl}
24\,\underline{)\,13750} & \text{gr.} \\
20\,\underline{)\,572} & \text{pwt.} + 22 \text{ gr.} \\
12\,\underline{)\,28} & \text{oz.} + 12 \text{ pwt.} \\
2 & \text{lb.} + 4 \text{ oz.}
\end{array}$$

2 lb. 4 oz. 12 pwt. 22 gr. Ans.

(19)

$$\begin{array}{rl}
100\,\underline{)\,4763254} & \text{li.} \\
80\,\underline{)\,47632} & + 54 \text{ li.} \\
595 & + 32 \text{ ch.}
\end{array}$$

595 mi. 32 ch. 54 li. Ans.

Explanation.—There are 100 links in 1 chain; hence, in 4,763,254 li. there are as many chains as 100 is contained times in 4,763,254 li., or 47,632 ch. and 54 li. remaining. Write the 54 li. as a remainder. There are 80 ch. in 1 mile; hence, in 47,632 ch. there are as many miles as 80 is contained times in 47,632 ch., or 595 mi. and 32 ch. remaining.

Then, in 4,763,254 li. there are 595 mi. 32 ch. 54 li.

(20) 1 7 2 8) 7 6 4 3 2 5 cu. in.
2 7) 4 4 2 + 549 cu. in.
1 6 cu. yd. + 10 cu. ft.

16 cu. yd. 10 cu. ft. 549 cu. in. Ans.

Explanation.—There are 1,728 cu. in. in 1 cubic foot; hence, in 764,325 cu. in. there are as many cubic feet as 1,728 is contained times in 764,325, or 442 cu. ft. and 549 cu. in. remaining. Write the 549 cu. in. as a remainder. There are 27 cu. ft. in 1 cubic yard; hence, in 442 cu. ft. there are as many cubic yards as 27 is contained times in 442 cu. ft., or 16 cu. yd. and 10 cu. ft. remaining. Then, in 764,325 cu. in. there are 16 cu. yd. 10 cu. ft. 549 cu. in.

(21) We must arrange the different terms in columns, taking care to have like denominations in the same column.

rd.	yd.	ft.	in.
2	2	2	3
	4	1	9
		2	7
3	$2\frac{1}{2}$	0	7

or, 3 rd. 2 yd. 2 ft. 1 in. Ans.

Explanation.—We begin to add at the right-hand column. $7+9+3 = 19$ in.; as 12 in. make 1 foot, 19 in. = 1 ft. and 7 in. Place the 7 in. in the inches column, and reserve the 1 ft. to add to the next column.

1 (reserved) $+2+1+2 = 6$ ft. Since 3 ft. make 1 yard, 6 ft. = 2 yd. and 0 ft. remaining. Place the cipher in the column of feet and reserve the 2 yd. for the next column.

2 (reserved) $+4+2 = 8$ yd. Since $5\frac{1}{2}$ yd. = 1 rd., 8 yd. = 1 rd. and $2\frac{1}{2}$ yd. Place $2\frac{1}{2}$ yd. in the yards column and reserve 1 rd. for the next column; 1 (reserved) $+2 = 3$ rd.

Ans. = 3 rd. $2\frac{1}{2}$ yd. 0 ft. 7 in.,
or 3 rd. 2 yd. 1 ft. 13 in.,
or 3 rd. 2 yd. 2 ft. 1 in.

(22) We write the compound numbers so that the units of the same denomination shall stand in the same column.

gal.	qt.	pt.	gi.
3	3	1	3
6	0	1	2
4	0	0	1
	8	5	0
16 gal.	3 qt.	0 pt.	2 gi.

Beginning to add with the lowest denomination, we find that the sum of the gills is 1 + 2 + 3 = 6. Since there are 4 gi. in 1 pint, in 6 gi. there are as many pints as 4 is contained times in 6, or 1 pt. and 2 gi. We place 2 gi. under the gills column and reserve the 1 pt. for the pints column; the sum of the pints is 1 (reserved) + 5 + 1 + 1 = 8. Since there are 2 pt. in 1 quart, in 8 pt. there are as many quarts as 2 is contained times in 8, or 4 qt. and 0 pt. We place the cipher under the column of pints and reserve the 4 for the quarts column. The sum of the quarts is 4 (reserved) + 8 + 3 = 15. Since there are 4 qt. in 1 gallon, in 15 qt. there are as many gallons as 4 is contained times in 15, or 3 gal. and 3 qt. remaining. We now place the 3 under the quarts column and reserve the 3 gal. for the gallons column. The sum of the gallons column is 3 (reserved) + 4 + 6 + 3 = 16 gal. Since we cannot reduce 16 gal. to any higher denomination, we have 16 gal. 3 qt. 0 pt. and 2 gi. for the answer.

(23) Reduce the grains, pennyweights, and ounces to higher denominations.

24) 240 gr. 20) 125 pwt. 12) 50 oz.
10 pwt. 6 oz. 5 pwt. 4 lb. 2 oz.

Then, 3 lb. + 4 lb. 2 oz. + 6 oz. 5 pwt. + 10 pwt. =

lb.	oz.	pwt.	
3			
4	2		
	6	5	
		10	
7 lb.	8 oz.	15 pwt.	Ans.

(24) Since "seconds" is the lowest denomination in this problem, we find their sum first, which is 11 + 29 + 25 + 30

deg.	min.	sec.
11	16	12
13	19	30
20	0	25
0	26	29
10	17	11
55°	19′	47″

+12, or 107 seconds. Since there are 60 seconds in 1 minute, in 107 seconds there are as many minutes as 60 is contained times in 107, or 1 minute and 47 seconds remaining. We place the 47 under the seconds column and reserve the 1 for the minutes column. The sum of the minutes is 1 (reserved) +17+26+19+16, or 79. Since there are 60 minutes in 1 degree, in 79 minutes there are as many degrees as 60 is contained times in 79, or 1 degree and 19 minutes remaining. We place the 19 under the minutes column and reserve the 1 degree for the degrees column. The sum of the degrees is 1 (reserved) +10+20+13+11, or 55 degrees. Since we cannot reduce 55 degrees to any higher denomination, we have 55° 19′ 47″ for the answer.

(25) Since "inches" is the lowest denomination in this problem, we find their sum first, which is 11+8+6, or 25 inches. Since there are 12 inches in 1 foot, in 25 inches there are as many feet as 12 is contained times in 25, or 2 feet and 1 inch remaining. Place the 1 inch under the inches column, and reserve the 2 feet to add to the column of feet. The sum of the feet is 2 feet (reserved) +2+1 = 5 feet. Since there are 3 feet in 1 yard, in 5 feet there are as many yards as 3 is contained times in 5 feet, or 1 yard and 2 feet remaining. Place the 2 feet under the column of feet, and reserve the 1 yard to add to the column of yards. The sum of the yards is 1 yard (reserved) +4 +5 = 10 yards. Since there are $5\frac{1}{2}$ yards in 1 rod, in 10 yards there are as many rods as $5\frac{1}{2}$ is contained times in 10, or 1 rod and $4\frac{1}{2}$ yards remaining. Place the $4\frac{1}{2}$ yards under

	rd.	yd.	ft.	in.	
	130	5	1	6	
	215	0	2	8	
	304	4	0	11	
	650	$4\frac{1}{2}$	2	1	
mi.	rd.	yd.	ft.	in.	
or, 2	10	5	0	7	Ans.

the column of yards, and reserve the 1 rod for the column of rods. The sum of the rods is 1 (reserved) $+304+215+130=650$ rods. Place 650 rods under the column of rods. Therefore, the sum is 650 rd. $4\frac{1}{2}$ yd. 2 ft. 1 in. Or, since $\frac{1}{2}$ a yard $=$ 1 ft. 6 in., and since there are 320 rods in 1 mile, the sum may be expressed as 2 mi. 10 rd. 5 yd. 0 ft. 7 in. Ans.

(26) Since "square links" is the lowest denomination in this problem, we find their sum first, which is $21+23+16+18+23+21$, or 122 square links. Place 122 square links under the column of square links. The sum of the square rods is $2+3+2+2+2+3$, or 14 square rods. Place 14 square rods under the column of square rods. The sum of the square chains is 323 square chains. Since there are 10 square chains in 1 acre, in 323 square chains there are as many acres as 10 is contained times in 323 square chains, or 32 acres and 3 square chains remaining. Place 3 square chains under the column of square chains, and reserve the 32 acres to add to the column of acres. The sum of the acres is 32 acres (reserved) $+46+25+56+47+28+21$, or 255 acres. Place 255 acres under the column of acres. Therefore, the sum is 255 A. 3 sq. ch. 14 sq. rd. 122 sq. li. Ans.

A.	sq. ch.	sq. rd.	sq. li.
21	67	3	21
28	78	2	23
47	6	2	18
56	59	2	16
25	38	3	23
46	75	2	21
255	3	14	122

(27) Before we can subtract 300 ft. from 20 rd. 2 yd. 2 ft. and 9 in., we must reduce the 300 ft. to higher denominations.

Since there are 3 feet in 1 yard, in 300 feet there are as many yards as 3 is contained times in 300, or 100 yards. There are $5\frac{1}{2}$ yards in 1 rod, hence in 100 yards there are as many rods as $5\frac{1}{2}$, or $\frac{11}{2}$, is contained times in $100 = 18\frac{2}{11}$ rods.

$$100\div\frac{11}{2}=100\times\frac{2}{11}=\frac{100\times 2}{11}=\frac{200}{11}\quad 11)\,2\,0\,0\,(\,1\,8\tfrac{2}{11}\text{ rd.}$$

$$\begin{array}{r} 1\,1 \\ \hline 9\,0 \\ 8\,8 \\ \hline 2 \end{array}$$

Since there are $5\frac{1}{2}$ or $\frac{11}{2}$ yards in 1 rod, in $\frac{2}{11}$ rod there are $\frac{\cancel{2}}{\cancel{11}}\times\frac{\cancel{11}}{\cancel{2}}$, or 1 yard, so we find that 300 feet equals 18 rods and 1 yard. The problem now is as follows: From 20 rd. 2 yd. 2 ft. and 9 in. take 18 rd. and 1 yd.

We place the smaller number under the larger one, so that the units of the same denomination fall in the same column. Beginning with the lowest denomination, we see that 0 inches from 9 inches leaves 9 inches. Going to the next higher denomination, we see that 0 feet from 2 feet leaves 2 feet. Subtracting 1 yard from 2 yards, we have 1 yard remaining, and 18 rods from 20 rods leaves 2 rods. Therefore, the difference is 2 rd. 1 yd. 2 ft. 9 in. Ans.

rd.	yd.	ft.	in.
20	2	2	9
18	1	0	0
2	1	2	9

(28)	A.	sq. rd.	sq. yd.	
	1 1 4	8 0	2 5	
	7 5	7 0	3 0	
	3 9	9	2 $5\frac{1}{4}$	Ans.

Explanation.—Place the subtrahend under the minuend so that like denominations are under each other. Then begin at the right with the lowest denomination. We cannot subtract 30 from 25, so we take 1 square rod (= $30\frac{1}{4}$ square yards) from 80 square rods, leaving 79 square rods; adding $30\frac{1}{4}$ square yards to 25 square yards, we have $55\frac{1}{4}$ square yards; subtracting 30 from $55\frac{1}{4}$ square yards, leaves $25\frac{1}{4}$ yards; we now subtract 70 square rods from 79 square rods, which leaves 9 square rods; next, we subtract 75 acres from

114 acres, which leaves 39 acres, which we place under the column of acres.

(29) If 10 gal. 2 qt. and 1 pt. of molasses are sold from a hogshead at one time, and 26 gal. 3 qt. are sold at another time, then the total amount of molasses sold equals 10 gal. 2 qt. 1 pt. plus 26 gal. 3 qt.

Since the pint is the lowest denomination, we add the pints first, which equal 0 + 1, or 1 pint. We cannot reduce 1 pint to any higher denomination, so we place it under the pints column. The number of quarts is 3 + 2, or 5. Since there are 4 quarts in 1 gallon, in 5 quarts there are as many gallons as 4 is contained times in 5, or 1 gallon and 1 quart remaining. We place the 1 quart under the quarts column, and reserve the 1 gallon to add to the column of gallons. The number of gallons equals 1 (reserved) + 26 + 10, or 37 gallons.

gal.	qt.	pt.
10	2	1
26	3	0
37	1	1

If 37 gal. 1 qt. and 1 pt. are sold from a hogshead of molasses (63 gal.), there remains the difference between 63 gal. and 37 gal. 1 qt. 1 pt., or 25 gal. 2 qt. 1 pt.

63 gal. is the same as 62 gal. 3 qt. 2 pt., since 1 gal. equals 4 qt., and 1 qt. equals 2 pt.

Beginning with the lowest denomination, we subtract 1 pt. from the 2 pt. 1 pint from 2 pints leaves 1 pint, 1 quart from 3 quarts leaves 2 quarts, and 37 gallons from 62 gallons leaves 25 gallons. Therefore, there are 25 gal. 2 qt. and 1 pt. of molasses remaining in the hogshead. Ans.

gal.	qt.	pt.
62	3	2
37	1	1
25	2	1

(30) If a person were born June 19, 1850, in order to find how old he would be on Aug. 3, 1892, subtract the earlier date from the later date.

On Aug. 3, 7 mo. and 3 da. have elapsed from the beginning of the year, and on June 19, 5 mo. and 19 da.

Beginning with the lowest denomination, we find that 19 days cannot be taken from 3 days, so we take 1 month from 7 months. The 1 month which we took equals 30 days, for in all cases 30 days are allowed to a month. Adding 30 days to the 3 days, we have 33 days; subtracting 19 days from 33 days, we have 14 days remaining. Since we borrowed 1 month from the months column, we have 7 — 1, or 6 months remaining; subtracting 5 months from 6 months, we have 1 month remaining. 1,850 from 1,892 leaves 42 years. Therefore, he would be 42 years, 1 month, and 14 days old. Ans.

yr.	mo.	da.
1892	7	3
1850	5	19
42	1	14

(31) If a note given Aug. 5, 1890, were paid June 3, 1892, in order to find the length of time it was due, subtract the earlier date from the later date.

Beginning with the lowest denomination, we find that 5 cannot be subtracted from 3, so we take a unit from the next higher denomination, which is months. The 1 month which we take equals 30 days. Adding the 30 days to the 3 days, we have 33 days. 5 days from 33 days leaves 28 days. Since we took 1 month from the months column, only 4 months remain. 7 months cannot be taken from 4 months, so we take 1 year from the years column, which equals 12 months. 12 months + 4 months = 16 months. 7 months from 16 months = 9 months. Since we took 1 year from the years column, we have 1,892 — 1, or 1,891 remaining. 1,890 from 1,891 leaves 1 year. Hence, the note ran 1 year, 9 months, and 28 days. Ans.

yr.	mo.	da.
1892	5	3
1890	7	5
1	9	28

(32) Write the number of the year, month, day, hour, and minute of the earlier date under the year, month, day, hour, and minute of the later date, and subtract.

22 minutes before 8 o'clock is the same as 38 minutes after 7 o'clock. 7 o'clock P. M. is 19 hours from the beginning of the day, as there are 12 hours in the morning and 7 in the

afternoon. December is 11 months from the beginning of the year.

10 o'clock A. M. is 10 hours from the beginning of the day. July is 6 months from the beginning of the year. The minuend would be the later date, or 1,888 years, 11 months, 11 days, 19 hours, and 38 minutes.

The subtrahend would be the earlier date or 1,883 years, 6 months, 3 days, 10 hours, and 16 minutes.

Subtracting, we have

yr.	mo.	da.	hr.	min.
1888	11	11	19	38
1883	6	3	10	16
5	5	8	9	22

or, 5 yr. 5 mo. 8 da. 9 hr. and 22 min. Ans.

16 minutes subtracted from 38 minutes leaves 22 minutes; 10 hours from 19 hours leaves 9 hours; 3 days from 11 days leaves 8 days; 6 months subtracted from 11 months leaves 5 months; 1,883 from 1,888 leaves 5 years.

(33) In multiplication of denominate numbers, we place the multiplier under the lowest denomination of the multiplicand, as

17 ft. 3 in.
51
879 ft. 9 in.

and begin at the right to multiply. $51 \times 3 = 153$ in. As there are 12 inches in 1 foot, in 153 in. there are as many feet as 12 is contained times in 153, or 12 feet and 9 inches remaining. Place the 9 inches under the inches, and reserve the 12 feet. 51×17 ft. $=$ 867 ft. 867 ft. $+$ 12 ft. (reserved) $=$ 879 ft.

879 feet can be reduced to higher denominations by dividing by 3 feet to find the number of yards, and by $5\frac{1}{2}$ yards to find the number of rods.

3)879 ft. 9 in.
5.5)293 yd.
53 rd. $1\frac{1}{2}$ yd.

Then, product = 53 rd. $1\frac{1}{2}$ yd. 0 ft. 9 in., or 53 rd. 1 yd. 2 ft. 3 in. Ans.

(34)

	qt.	pt.	gi.	
	3	1	3	
			4.7	
	1 8.2	0	.1	
or,	1 8 qt.	0 pt.	1.7 gi.	
or, 4 gal.	2 qt.	0 pt.	1.7 gi.	Ans.

Place the multiplier under the lowest denomination of the multiplicand, and proceed to multiply. 4.7 × 3 gi. = 14.1 gi. As 4 gi. = 1 pt., there are as many pints in 14.1 gi. as 4 is contained times in 14.1 = 3.5 pt. and .1 gi. over. Place .1 under gills and carry the 3.5 pt. forward. 4.7 × 1 pt. = 4.7 pt.; 4.7 + 3.5 pt. = 8.2 pt. As 2 pt. = 1 qt., there are as many quarts in 8.2 pt. as 2 is contained times in 8.2 = 4.1 qt. and no pints over. Place a cipher under the pints, and carry the 4.1 qt. to the next product. 4.7 × 3 qt. = 14.1; 14.1 + 4.1 qt. = 18.2 qt. The answer now is 18.2 qt. 0 pt. .1 gi. Reducing the fractional part of a quart, we have 18 qt. 0 pt. 1.7 gi. (.2 qt. = .2 × 8 = 1.6 gi.; 1.6 + .1 gi. = 1.7 gi.). Then we can reduce 18 qt. to gallons (18 ÷ 4 = 4 gal. and 2 qt.) = 4 gal. 2 qt. 1.7 gi. Ans.

The answer may be obtained in another and much easier way by reducing all to gills, multiplying by 4.7, and then changing back to quarts and pints. Thus,

3 qt.
× 2 pt.
6 pt.
+ 1 pt.
7 pt.
× 4 gi.
2 8 gi.
+ 3 gi.
3 1 gi.

3 qt. 1 pt. 3 gi. = 31 gi.
31 gi. × 4.7 = 145.7 gi.

4) 1 4 5.7 gi.
2) 3 6 pt. + 1.7 gi.
1 8 qt. + 0 pt.

Ans. = 18 qt. 1.7 gi.;
or, 4 gal. 2 qt. 1.7 gi.

(35) (3 lb. 10 oz. 13 pwt. 12 gr.)×1.5 = ?

$$\begin{array}{rl}
 & 3\text{ lb. }10\text{ oz. }13\text{ pwt. }12\text{ gr.}\\
\times & \underline{12}\\
 & 36\text{ oz.}\\
+ & \underline{10}\\
 & 46\text{ oz.}\\
\times & \underline{20}\\
 & 920\text{ pwt.}\\
+ & \underline{13}\\
 & 933\text{ pwt.}\\
\times & \underline{24}\\
 & 22392\text{ gr.}\\
+ & \underline{12}\\
 & 22404\text{ gr.}
\end{array}$$

$$22{,}404\text{ gr.}\times 1.5 = 33{,}606\text{ gr.}$$

$$\begin{array}{r}
24\,\underline{)\,33606}\text{ gr.}\\
20\,\underline{)\,1400}\text{ pwt.}+6\text{ gr.}\\
12\,\underline{)\,70}\text{ oz.}+0\text{ pwt.}\\
5\text{ lb.}+10\text{ oz.}
\end{array}$$

Since there are 24 gr. in 1 pwt., in 33,606 gr. there are as many pennyweights as 24 is contained times in 33,606, or 1,400 pwt. and 6 gr. remaining. This gives us the number of grains in the answer. We now reduce 1,400 pwt. to higher denominations. Since there are 20 pwt. in 1 oz., in 1,400 pwt. there are as many ounces as 20 is contained times in 1,400, or 70 oz. and 0 pwt. remaining; therefore, there are 0 pwt. in the answer. We reduce 70 oz. to higher denominations. Since there are 12 oz. in 1 lb., in 70 oz. there are as many pounds as 12 is contained times in 70, or 5 lb. and 10 oz. remaining. We cannot reduce 5 lb. to any higher denominations. Therefore, our answer is 5 lb. 10 oz. 6 gr.

Another but more complicated way of working this problem is as follows:

lb.	oz.	pwt.	gr.	
3	10	13	12	
			1.5	
4.5	15	19.5	18	
or, 4	21	19	30	
or, 5	10	0	6	Ans.

To get rid of the decimal in the pounds, reduce .5 of a pound to ounces. Since 1 lb. = 12 oz., .5 of a pound equals .5 lb. × 12 = 6 oz. 6 oz. + 15 oz. = 21 oz. We now have 4 lb. 21 oz. 19.5 pwt. and 18 gr.; but we still have a decimal in the column of pwt., so we reduce .5 pwt. to grains to get rid of it. Since 1 pwt. = 24 gr., .5 pwt. = .5 pwt. × 24 = 12 gr. 12 gr. + 18 gr. = 30 gr. We now have 4 lb. 21 oz. 19 pwt. and 30 gr. Since there are 24 gr. in 1 pwt., in 30 gr. there is 1 pwt. and 6 gr. remaining. Place 6 gr. under the column of grains and add 1 pwt. to the pwt. column. Adding 1 pwt., we have 19 + 1 = 20 pwt. Since there are 20 pwt. in 1 oz., we have 1 oz. and 0 pwt. remaining. Write the 0 pwt. under the pwt. column, and reserve the 1 oz. to the oz. column. 21 oz. + 1 oz. = 22 oz. Since there are 12 oz. in 1 lb., in 22 oz. there is 1 lb. and 10 oz. remaining. Write the 10 oz. under the ounce column, and reserve the 1 lb. to add to the pounds column. 4 lb. + 1 lb. (reserved) = 5 lb. Hence, the answer equals 5 lb. 10 oz. 6 gr.

(36) If each barrel of apples contains 2 bu. 3 pk. and 6 qt., then 9 bbl. will contain 9 × (2 bu. 3 pk. 6 qt.).

bu.	pk.	qt.
2	3	6
		9
18	27	54
or, 26	1	6

We write the multiplier under the lowest denomination of the multiplicand, which is quarts in this problem. 9 times 6 qt. equals 54 qt. There are 8 qt. in 1 pk., and in 54 qt. there are as many pecks as 8 is contained times in 54, or 6 pk. and 6 qt. We write the 6 qt. under the column of quarts, and reserve the 6 pk. to add to the product of the pecks. 9 times 3 pk. equals 27 pk.; 27 pk. plus the 6 pk. (reserved) equals 33 pk. Since there are 4 pk. in 1 bu., in 33 pk. there are as many bushels as 4 is contained times in 33, or 8 bu. and 1 pk. remaining. We write the 1 pk. under the column of pecks, and reserve the 8 bu.

for the product of the bushels. 9 times 2 bu. plus the 8 bu. (reserved) equals 26 bu. Therefore, we find that 9 bbl. contain 26 bu. 1 pk. 6 qt. of apples. Ans.

(37) (7 T. 15 cwt. 10.5 lb.)×1.7 = ? When the multiplier is a decimal, instead of multiplying the denominate numbers as in the case when the multiplier is a whole number, it is much easier to reduce the denominate numbers to the lowest denomination given; then multiply that result by the decimal, and, lastly, reduce the product to higher denominations. Although the correct answer can be obtained by working examples involving decimals in the manner as in the last example, it is much more complicated than this method.

$$\begin{array}{rl}
 & 7\text{ T. }15\text{ cwt. }10.5\text{ lb.} \\
\times & \underline{20} \\
 & 140\text{ cwt.} \\
+ & \underline{15} \\
 & 155\text{ cwt.} \\
\times & \underline{100} \\
 & 15500\text{ lb.} \\
+ & \underline{10.5} \\
 & 15510.5\text{ lb.}
\end{array}$$

15,510.5 lb.×1.7 = 26,367.85 lb.

There are 100 lb. in 1 cwt., and in 26,367.85 lb. there are as many cwt. as 100 is contained times in 26,367.85, which equals 263 cwt. and 67.85 lb. remaining. Since we have the number of pounds for our answer, we reduce 263 cwt. to higher denominations. There are 20 cwt. in 1 ton, and in 263 cwt. there are as many tons as 20 is contained times in 263, or 13 T. and 3 cwt. remaining. Since we cannot reduce 13 T. any higher, our answer is 13 T. 3 cwt. 67.85 lb. Or, since .85 lb. = .85 lb.×16 = 13.6 oz., the answer may be written 13 T. 3 cwt. 67 lb. 13.6 oz.

$$\begin{array}{r}
100\,\underline{)\,26367.85\text{ lb.}} \\
20\,\underline{)\,263}\text{ cwt.} + 67.85\text{ lb.} \\
13\text{ T.} + 3\text{ cwt.}
\end{array}$$

(38) $7\,\underline{)\,358\text{ A. }\ 57\text{ sq. rd. }\ 6\text{ sq. yd. }\ 2\text{ sq. ft.}}$

51 A. 31 sq. rd. 0 sq. yd. 8 sq. ft. Ans.

We begin with the highest denomination, and divide each term in succession by 7.

7 is contained in 358 A. 51 times and 1 A. remaining. We write the 51 A. under the 358 A. and reduce the remaining 1 A. to square rods = 160 sq. rd.; 160 sq. rd. + the 57 sq. rd. in the dividend = 217 sq. rd. 7 is contained in 217 sq. rd. 31 times and 0 sq. rd. remaining. 7 is not contained in 6 sq. yd., so we write 0 under the sq. yd. and reduce 6 sq. yd. to square feet. 9 sq. ft. × 6 = 54 sq. ft. 54 sq. ft. + 2 sq. ft. in the dividend = 56 sq. ft. 7 is contained in 56 sq. ft. 8 times. We write 8 under the 2 sq. ft. in the dividend.

(39)

$$12 \,\underline{)\; 282 \text{ bu.} \quad 3 \text{ pk.} \quad 1 \text{ qt.} \quad 1 \text{ pt.}}$$
$$23 \text{ bu.} \quad 2 \text{ pk.} \quad 2 \text{ qt.} \quad \tfrac{1}{4} \text{ pt.} \quad \text{Ans.}$$

12 is contained in 282 bu. 23 times and 6 bu. remaining. We write 23 bu. under the 282 bu. in the dividend, and reduce the remaining 6 bu. to pecks = 24 pk. + the 3 pk. in the dividend = 27 pk. 12 is contained in 27 pk. 2 times and 3 pk. remaining. We write 2 pk. under the 3 pk. in the dividend, and reduce the remaining 3 pk. to quarts. 3 pk. = 24 qt.; 24 qt. + the 1 qt. in the dividend = 25 qt. 12 is contained in 25 qt. 2 times and 1 qt. remaining. We write 2 qt. under the 1 qt. in the dividend, and reduce 1 qt. to pints = 2 pt. + the 1 pt. in the dividend = 3 pt. 3 ÷ 12 = $\frac{3}{12}$ or $\frac{1}{4}$ pt.

(40) We must first reduce 23 miles to feet before we can divide by 30 feet. 1 mile contains 5,280 ft.; hence 23 mi. contain 5,280 × 23 = 121,440 ft.

121,440 ft. ÷ 30 ft. = 4,048 rails for 1 side of the track.

The number of rails for 2 sides of the track = 2 × 4,048, or 8,096 rails. Ans.

(41) In this case, where both dividend and divisor are compound, reduce each to the lowest denomination mentioned in either, and then divide as in simple numbers.

$$\begin{array}{rl} 1 & \text{bu. } 1 \text{ pk. } 7 \text{ qt.} \\ \times\ 4 & \\ \hline 4 & \text{pk.} \\ +\ 1 & \\ \hline 5 & \text{pk.} \\ \times\ 8 & \\ \hline 40 & \text{qt.} \\ +\ 7 & \\ \hline 47 & \text{qt.} \end{array} \qquad \begin{array}{rl} 356 & \text{bu. } 3 \text{ pk. } 5 \text{ qt.} \\ \times\ 4 & \\ \hline 1424 & \text{pk.} \\ +\ 3 & \\ \hline 1427 & \text{pk.} \\ \times\ 8 & \\ \hline 11416 & \text{qt.} \\ +\ 5 & \\ \hline 11421 & \text{qt.} \end{array}$$

$$\begin{array}{r} 47)11421(243 \\ 94 \\ \hline 202 \\ 188 \\ \hline 141 \\ 141 \\ \hline \end{array}$$

11,421 qt. ÷ 47 qt. = 243 boxes. Ans.

(42) We must first reduce 16 square miles to acres. In 1 sq. mi. there are 640 A., and in 16 sq. mi. there are 16×640 A. = 10,240 A.

$$\begin{array}{l} 62\overline{)10240 \text{ A.}} \\ \hline \quad 165 \text{ A. } 25 \text{ sq. rd. } 24 \text{ sq. yd. } 3 \text{ sq. ft. } 80+ \text{ sq. in.} \end{array}$$

Ans.

62 is contained in 10,240 A. 165 times and 10 A. remaining. We write 165 A. under the 10,240 A. in the dividend and reduce 10 A. to sq. rd. In 1 A. there are 160 sq. rd., and in 10 A. there are 10×160 = 1,600 sq. rd. 62 is contained in 1,600 sq. rd. 25 times and 50 sq. rd. remaining. We write 25 sq. rd. in the quotient and reduce 50 sq. rd. to sq. yd. In 1 sq. rd. there are $30\frac{1}{4}$ sq. yd., and in 50 sq. rd. there are 50 × $30\frac{1}{4}$ sq. yd. = $1{,}512\frac{1}{2}$ sq. yd. 62 is contained in $1{,}512\frac{1}{2}$ sq. yd. 24 times and $24\frac{1}{2}$ sq. yd. remaining. We write 24 sq. yd. in the quotient and reduce $24\frac{1}{2}$ sq. yd. to sq. ft. In 1 sq. yd. there are 9 sq. ft., and in $24\frac{1}{2}$ sq. yd. there are $24\frac{1}{2}\times 9 = 220\frac{1}{2}$ sq. ft. 62 is contained in $220\frac{1}{2}$ sq. ft. 3 times and $34\frac{1}{2}$ sq. ft. remaining. We write 3 sq. ft. in the quotient and reduce $34\frac{1}{2}$ sq. ft. to sq. in. In 1 sq. ft. there are 144 sq. in., and in $34\frac{1}{2}$ sq. ft. there are $34\frac{1}{2}\times 144 = 4{,}968$ sq. in.

62 is contained in 4,968 sq. in. 80 times and 8 sq. in. remaining. We write 80+ sq. in. in the quotient.

It should be borne in mind that it is only for the purpose of illustrating the method that this problem is carried out to square inches. It is not customary to reduce any lower than square rods in calculating the area of a farm.

(43) To square a number, we must multiply the number by itself once; that is, use the number twice as a factor. Thus, the second power of 108 is 108 × 108 = 11,664. Ans.

```
  108
  108
  ---
  864
108
-----
11664
```

(44) The third power of 181.25 equals the number obtained by using 181.25 as a factor three times. Thus, the third power of 181.25 is 181.25 × 181.25 × 181.25 = 5,954,345.703125. Ans.

```
        181.25
        181.25
        ------
         90625
        36250
       18125
     145000
     18125
    ----------
    32851.5625
        181.25
    ----------
    1642578125
    657031250
   328515625
 2628125000
 328515625
--------------
5954345.703125
```

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are 2 + 2 = 4 decimal places in the first product.

Since there are 4 decimal places in the multiplicand and 2 in the multiplier, there are 4 + 2 = 6 decimal places in the final product.

(45) The fourth power of 27.61 is the number obtained by using 27.61 as a factor four times. Thus, the fourth power of 27.61 is 27.61 × 27.61 × 27.61 × 27.61 = 581,119.73780641. Ans.

```
                    2 7.6 1
                    2 7.6 1
              -------------
                    2 7 6 1
                1 6 5 6 6
              1 9 3 2 7
              5 5 2 2
              -------------
              7 6 2.3 1 2 1
                    2 7.6 1
              -------------
              7 6 2 3 1 2 1
          4 5 7 3 8 7 2 6
        5 3 3 6 1 8 4 7
      1 5 2 4 6 2 4 2
      ---------------------
      2 1 0 4 7.4 3 7 0 8 1
                    2 7.6 1
      ---------------------
      2 1 0 4 7 4 3 7 0 8 1
  1 2 6 2 8 4 6 2 2 4 8 6
1 4 7 3 3 2 0 5 9 5 6 7
4 2 0 9 4 8 7 4 1 6 2
---------------------------
5 8 1 1 1 9.7 3 7 8 0 6 4 1
```

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are $2+2 = 4$ decimal places in the first product.

Since there are 4 decimal places in the multiplicand and 2 in the multiplier, there are $4+2 = 6$ decimal places in the second product.

Since there are 6 decimal places in the multiplicand and 2 in the multiplier, there are $6+2 = 8$ decimal places in the final product.

(46) (*a*) $106^2 = 106 \times 106 = 11{,}236$. Ans.

```
    1 0 6
    1 0 6
    -----
    6 3 6
1 0 6 0
---------
1 1 2 3 6
```

(*b*) $(182\frac{1}{8})^2 = 182\frac{1}{8} \times 182\frac{1}{8} = 33{,}169.515625$. Ans.

$\frac{1}{8} = 8\,)\,\underline{1.000}$

.125

```
          1 8 2.1 2 5
          1 8 2.1 2 5
          -----------
          9 1 0 6 2 5
        3 6 4 2 5 0
      1 8 2 1 2 5
    3 6 4 2 5 0
1 4 5 7 0 0 0
1 8 2 1 2 5
---------------------
3 3 1 6 9.5 1 5 6 2 5
```

Since there are 3 decimal places in the multiplier and 3 in the multiplicand, there are $3+3 = 6$ decimal places in the product.

(*c*) $.005^2 = .005 \times .005 = .000025$. Ans.

$$\begin{array}{r} .005 \\ .005 \\ \hline .000025 \end{array}$$

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, there are $3+3=6$ decimal places in the product.

(*d*) $.0063^2 = .0063 \times .0063 = .00003969$. Ans.

$$\begin{array}{r} .0063 \\ .0063 \\ \hline 189 \\ 378 \\ \hline .00003969 \end{array}$$

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, there are $4+4=8$ decimal places in the product.

(*e*) $10.06^2 = 10.06 \times 10.06 = 101.2036$. Ans.

$$\begin{array}{r} 10.06 \\ 10.06 \\ \hline 6036 \\ 100600 \\ \hline 101.2036 \end{array}$$

Since there are 2 decimal places in the multiplicand and 2 in the multiplier, there are $2+2=4$ decimal places in the product.

(47) (*a*) $753^3 = 753 \times 753 \times 753 = 426,957,777$. Ans.

$$\begin{array}{r} 753 \\ 753 \\ \hline 2259 \\ 3765 \\ 5271 \\ \hline 567009 \\ 753 \\ \hline 1701027 \\ 2835045 \\ 3969063 \\ \hline 426957777 \end{array}$$

(*b*) $987.4^3 = 987.4 \times 987.4 \times 987.4 = 962,674,279.624$.
Ans.

$$\begin{array}{r}
9\,8\,7.4 \\
9\,8\,7.4 \\
\hline
3\,9\,4\,9\,6 \\
6\,9\,1\,1\,8 \\
7\,8\,9\,9\,2 \\
8\,8\,8\,6\,6 \\
\hline
9\,7\,4\,9\,5\,8.7\,6 \\
9\,8\,7.4 \\
\hline
3\,8\,9\,9\,8\,3\,5\,0\,4 \\
6\,8\,2\,4\,7\,1\,1\,3\,2 \\
7\,7\,9\,9\,6\,7\,0\,0\,8 \\
8\,7\,7\,4\,6\,2\,8\,8\,4 \\
\hline
9\,6\,2\,6\,7\,4\,2\,7\,9.6\,2\,4
\end{array}$$

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, there are $1+1 = 2$ decimal places in the first product.

Since there are 2 decimal places in the multiplicand and 1 in the multiplier, there are $2+1 = 3$ decimal places in the final product.

(*c*) $.005^3 = .005 \times .005 \times .005 = .000000125$. Ans.

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, there are $3+3 = 6$ decimal places in the first product; but, as there are only 2 figures in the product, we prefix 4 ciphers to make the 6 decimal places.

$$\begin{array}{r}
.0\,0\,5 \\
.0\,0\,5 \\
\hline
.0\,0\,0\,0\,2\,5 \\
.0\,0\,5 \\
\hline
.0\,0\,0\,0\,0\,0\,1\,2\,5
\end{array}$$

Since there are 6 decimal places in the multiplicand and 3 in the multiplier, there are $6+3 = 9$ decimal places in the final product. In this case we prefix 6 ciphers to form the 9 decimal places.

(*d*) $.4044^3 = .4044 \times .4044 \times .4044 = .066135317184$. Ans.

$$\begin{array}{r}
.4\,0\,4\,4 \\
.4\,0\,4\,4 \\
\hline
1\,6\,1\,7\,6 \\
1\,6\,1\,7\,6 \\
1\,6\,1\,7\,6\,0 \\
\hline
.1\,6\,3\,5\,3\,9\,3\,6 \\
.4\,0\,4\,4 \\
\hline
6\,5\,4\,1\,5\,7\,4\,4 \\
6\,5\,4\,1\,5\,7\,4\,4 \\
6\,5\,4\,1\,5\,7\,4\,4\,0 \\
\hline
.0\,6\,6\,1\,3\,5\,3\,1\,7\,1\,8\,4
\end{array}$$

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, there are $4+4 = 8$ decimal places in the first product.

Since there are 8 decimal places in the second multiplicand and 4 in the multiplier, there are $8+4 = 12$ decimal places in the final product; but, as there are only 11 figures in the product, we prefix 1 cipher to make 12 decimal places.

(48) $2^5 = 2\times2\times2\times2\times2 = 32$. Ans.

(49) $3^4 = 3\times3\times3\times3 = 81$. Ans.

(50) (*a*) $67.85^2 = 67.85\times67.85 = 4{,}603.6225$. Ans.

$$\begin{array}{r}
67.85\\
67.85\\
\hline
33925\\
54280\\
47495\\
40710\\
\hline
4603.6225
\end{array}$$

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are $2+2 = 4$ decimal places in the product.

(*b*) $967{,}845^2 = 967{,}845\times967{,}845 = 936{,}723{,}944{,}025$. Ans.

$$\begin{array}{r}
967845\\
967845\\
\hline
4839225\\
3871380\\
7742760\\
6774915\\
5807070\\
8710605\\
\hline
936723944025
\end{array}$$

(*c*) A fraction may be raised to any power by raising both numerator and denominator to the required term.

Thus, $\left(\frac{3}{8}\right)^2 = \frac{3}{8}\times\frac{3}{8} = \frac{3\times3}{8\times8} = \frac{9}{64}$. Ans.

(*d*) $\left(\frac{1}{4}\right)^2 = \frac{1}{4}\times\frac{1}{4} = \frac{1\times1}{4\times4} = \frac{1}{16}$. Ans.

(51) (*a*) $5^{10} = 5\times5\times5\times5\times5\times5\times5\times5\times5\times5 = 9{,}765{,}625$. Ans.

(*b*) $9^5 = 9\times9\times9\times9\times9 = 59{,}049$. Ans.

(*a*)

$$\begin{array}{r} 5\\ 5\\ \hline 25\\ 5\\ \hline 125\\ 5\\ \hline 625\\ 5\\ \hline 3125\\ 5\\ \hline 15625\\ 5\\ \hline 78125\\ 5\\ \hline 390625\\ 5\\ \hline 1953125\\ 5\\ \hline 9765625 \end{array}$$

(*b*)

$$\begin{array}{r} 9\\ 9\\ \hline 81\\ 9\\ \hline 729\\ 9\\ \hline 6561\\ 9\\ \hline 59049 \end{array}$$

(52) (*a*) $1.2^4 = 1.2\times1.2\times1.2\times1.2 = 2.0736$. Ans.

$$\begin{array}{r} 1.2\\ 1.2\\ \hline 24\\ 12\\ \hline 1.44\\ 1.2\\ \hline 288\\ 144\\ \hline 1.728\\ 1.2\\ \hline 3456\\ 1728\\ \hline 2.0736 \end{array}$$

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, we must point off 1 + 1 = 2 decimal places in the first product.

Since there are 2 decimal places in the second multiplicand and 1 in the multiplier, we must point off 2 + 1 = 3 decimal places in the second product.

Since there are 3 decimal places in the third multiplicand and 1 in the multiplier, we must point off 3 + 1 = 4 decimal places in the final product.

(*b*) $11^6 = 11 \times 11 \times 11 \times 11 \times 11 \times 11 = 1,771,561.$ Ans.

$$\begin{array}{r}
11 \\
11 \\ \hline
121 \\
11 \\ \hline
1331 \\
11 \\ \hline
14641 \\
11 \\ \hline
161051 \\
11 \\ \hline
1771561
\end{array}$$

(*c*) $1^7 = 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1.$ Ans.

(*d*) $.01^4 = .01 \times .01 \times .01 \times .01 = .00000001.$ Ans.

Since there are 2 decimal places in the multiplicand and 2 in the multiplier, we must point off $2 + 2 = 4$ decimal places in the first product; but, as there is only 1 figure in the product, we prefix 3 ciphers to make the 4 necessary decimal places.

$$\begin{array}{r}
.01 \\
.01 \\ \hline
.0001 \\
.01 \\ \hline
.000001 \\
.01 \\ \hline
.00000001
\end{array}$$

Since there are 4 decimal places in the second multiplicand and 2 in the multiplier, we must point off $4 + 2 = 6$ decimal places in the second product. It is necessary to prefix 5 ciphers to make 6 decimal places.

Since there are 6 decimal places in the third multiplicand and 2 in the multiplier, we must point off $6 + 2 = 8$ decimal places in the product. It is necessary to prefix 7 ciphers to make 8 decimal places in the final product.

(*e*) $.1^5 = .1 \times .1 \times .1 \times .1 \times .1 = .00001.$ Ans.

Since there is 1 decimal place in the multiplicand and 1 in the multiplier, we must point off $1 + 1 = 2$ decimal places

$$\begin{array}{r} .1 \\ \underline{.1} \\ .01 \\ \underline{.1} \\ .001 \\ \underline{.1} \\ .0001 \\ \underline{.1} \\ .00001 \end{array}$$

in the first product. It is necessary to prefix 1 cipher to the product.

Since there are 2 decimal places in the second multiplicand and 1 in the multiplier, we must point off $2+1 = 3$ decimal places in the second product. It is necessary to prefix 2 ciphers to the second product.

Since there are 3 decimal places in the third multiplicand and 1 in the multiplier, we must point off $3+1 = 4$ decimal places in the third product. It is necessary to prefix 3 ciphers to this product.

Since there are 4 decimal places in the fourth multiplicand and 1 in the multiplier, we must point off $4+1 = 5$ decimal places in the final product. It is necessary to prefix 4 ciphers to this product.

(53) (*a*) $.0133^3 = .0133 \times .0133 \times .0133 = .000002352637$. Ans.

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, we must point off $4+4 = 8$ decimal places in the product; but, as there are only 5 figures in the product, we prefix 3 ciphers to form the 8 necessary decimal places in the first product.

$$\begin{array}{r} .0133 \\ \underline{.0133} \\ 399 \\ 399 \\ \underline{133} \\ .00017689 \\ \underline{.0133} \\ 53067 \\ 53067 \\ \underline{17689} \\ .000002352637 \end{array}$$

Since there are 8 decimal places in the multiplicand and 4 in the multiplier, we must point off $8+4 = 12$ decimal places in the product; but, as there are only 7 figures in the product, we prefix 5 ciphers to make the 12 necessary decimal places in the final product.

(*b*) $301.011^3 = 301.011 \times 301.011 \times 301.011 = 27{,}273{,}890.942264331$. Ans.

$$\begin{array}{r}
301.011\\
301.011\\
\hline
301011\\
301011\\
3010110\\
9030330\\
\hline
90607.622121\\
301.011\\
\hline
90607622121\\
90607622121\\
906076221210\\
2718228663630\\
\hline
27273890.942264331
\end{array}$$

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, we must point off $3+3 = 6$ decimal places in the first product.

Since there are 6 decimal places in the multiplicand and 3 in the multiplier, we must point off $6+3 = 9$ decimal places in the final product.

(*c*) $(\frac{1}{8})^3 = \frac{1}{8}\times\frac{1}{8}\times\frac{1}{8} = \frac{1\times1\times1}{8\times8\times8} = \frac{1}{512}$. Ans.

(*d*) To find any power of a mixed number, first reduce it to an improper fraction, and then multiply the numerators together for the numerator of the answer, and multiply the denominators together for the denominator of the answer.

$$(3\tfrac{3}{4})^3 = \tfrac{15}{4}\times\tfrac{15}{4}\times\tfrac{15}{4} = \frac{15\times15\times15}{4\times4\times4} = \frac{3,375}{64} = 52.734+. \quad \text{Ans.}$$

$$3\tfrac{3}{4} = \frac{3\times4+3}{4} = \frac{12+3}{4} = \tfrac{15}{4}.$$

$$\begin{array}{r}
15\\
15\\
\hline
75\\
15\\
\hline
225\\
15\\
\hline
1125\\
225\\
\hline
3375
\end{array}
\qquad
\begin{array}{rl}
64)\,3375.000 & (52.734+.\\
320 & \\
\hline
175 & \\
128 & \\
\hline
470 & \\
448 & \\
\hline
220 & \\
192 & \\
\hline
280 & \\
256 & \\
\hline
24 &
\end{array}$$

Since *three* ciphers were annexed to the dividend, *three* decimal places must be pointed off in the quotient. It is easy to see that the next figure will be a 3; hence, write the sign +, as shown.

(54) Evolution is the reverse of involution. In involution we find the *power* of a number by multiplying the number by itself one or more times, while in evolution we find the *number* or *root* which was multiplied by itself one or more times to make the power.

(55) (*a*)

1	$\sqrt{3'4\,8'6\,7'8\,4.4\,0'1\,0} = 1{,}867.29+$. Ans.
1	1
20	248
8	224
28	2467
8	2196
360	27184
6	26089
366	3734) 1095.000 (.293 or .29+
6	7468
3720	34820
7	33606
3727	12140
7	
3734	

Explanation.—Applying the short method described in Art. **91,** we extract the root by the regular method to four figures, since there are six figures in the answer, and $6 \div 2 + 1 = 4$. The last remainder is 1,095, and the last trial divisor (with the cipher omitted) is 3,734. Dividing 1,095 by 3,734, as shown, the quotient, is .293+, or .29+, using two figures. Annexing to the root, gives 1,867.29+. Ans.

```
(b) (a) 3             √9'0 0'0 0'9 9.4 0'0 9'0 0 = 3,000.0165+.
        3         (b)  9                                    Ans.
    (d) 6 0       (c)    0 0 0 0 9 9 4 0 0 9
          0                      6 0 0 0 0 1
        6 0 0                    3 9 4 0 0 8 0 0
            0                    3 6 0 0 0 1 5 6
        6 0 0 0                    3 4 0 0 6 4 4
              0
        6 0 0 0 0
                0
        6 0 0 0 0 0
                  1
        6 0 0 0 0 1
                  1
        6 0 0 0 0 2 0
                    6
        6 0 0 0 0 2 6
```

Explanation.—Beginning at the decimal point, we point the whole number into periods of two figures each, proceeding from right to left; also, point off the decimal into periods of two figures each, proceeding from left to right. The largest number whose square is contained in the first period, 9, is 3; hence, 3 is the first figure of the root. Place 3 at the left, as shown at (*a*), and multiply it by the first figure in the root, or 3. The result is 9. Write 9 under the first period, 9, as at (*b*), subtract, and there is no remainder. Bring down the next period, which is 00, as shown at (*c*). Add the root already found to the 3 at (*a*), obtaining 6, and annex a cipher to this 6, thus making it 60, which is the trial divisor, as shown at (*d*). Divide the dividend (*c*) by the trial divisor, and obtain 0 as the next figure in the root. Write 0 in the root, as shown, and also add it to the trial divisor, 60, and annex a cipher, thereby making the next trial divisor 600. Bring down the next period, 00, annex it to the dividend already obtained, and divide it by the trial divisor. 600 is contained in 0000, 0 times, so we place another cipher in the root. Write 0 in the root, as shown, and also add it to the trial divisor, 600, and annex a cipher, thereby making the next trial divisor 6,000. Bring down the next period, 99. The trial divisor, 6,000, is contained in 000099, 0 times, so we place 0 as the next figure in the root, as shown, and also add it to the trial divisor, 6,000, and annex a cipher, thereby making the next trial divisor 60,000. Bring down the next period, 40, and annex it to the dividend already obtained to form the new dividend, 00009940, and divide it by the trial divisor, 60,000. 60,000 is contained in 00009940, 0 times, so we place another cipher in the root, as shown, and also add it to the trial divisor, 60,000, and annex one cipher, thereby making the next trial divisor 600,000. Bring down the next period, 09, and annex it to the dividend already obtained to form the new dividend, 0000994009, and divide it by the trial divisor, 600,000. 600,000 is contained in 0000994009, once, so we place 1 as the next figure in the root, and also add it to the trial divisor, 600,000, thereby making the complete divisor 600,001. Multiply the complete divisor, 600,001, by 1, the sixth figure in the root, and subtract the result obtained from the dividend. The remainder is 394,008, to which we annex the next period, 00, to form the next new dividend, or 39,400,800. Add the sixth figure of the root, or 1, to the divisor, 600,001, and annex a cipher, thus obtaining 6,000,020 as the next trial divisor. Dividing 39,400,800 by 6,000,020, we find 6 to be the next figure of the root. Adding this last figure, 6, to the trial divisor, we obtain 6,000,026 for our next complete divisor, which multiplied by the last figure of the root, or 6, gives 36,000,156, which write under 39,400,800 and subtract. Since there is a remainder, it is clearly evident that the given power is not a perfect square, so we place + after the root. Since the next figure is 5, the answer is 3,000.017–.

In this problem there are seven periods—four in the whole number and three in the decimal—hence, there will be seven figures in the root, four figures constituting the whole number and three figures the decimal of the root. Hence, $\sqrt{9,000,099.4009} = 3,000.017-$.

(*c*)

$$\begin{array}{r l}
3 & \sqrt{.0\,0'1\,2'2\,5} = .035. \quad \text{Ans.}\\
\underline{3} & \underline{0\,0}\\
6\,0 & \quad 1\,2\\
\underline{5} & \quad \underline{9}\\
6\,5 & \quad 3\,2\,5\\
 & \quad \underline{3\,2\,5}
\end{array}$$

Pointing off periods, we find that the first period is composed of ciphers; hence, the first figure of the root will be a cipher. No further explanation is necessary, since this problem is solved in a manner exactly similar to the problem solved in Art. **83,** *Arithmetic*, § 2. Since there are three decimal periods in the power, there will be three decimal figures in the root.

(56) (*a*)

$$\begin{array}{r l}
1 & \sqrt{1'0\,7'9\,5.2\,1} = 103.9. \quad \text{Ans.}\\
\underline{1} & \underline{1}\\
2\,0 & \quad 0\,7\,9\,5\\
\underline{0} & \quad \underline{6\,0\,9}\\
2\,0\,0 & \quad 1\,8\,6\,2\,1\\
\underline{3} & \quad \underline{1\,8\,6\,2\,1}\\
2\,0\,3 & \\
\underline{3} & \\
2\,0\,6\,0 & \\
\underline{9} & \\
2\,0\,6\,9 &
\end{array}$$

(*b*)

$$\begin{array}{r l}
2 & \sqrt{7'3\,0'0\,8.0\,5} = 270.2+. \quad \text{Ans.}\\
\underline{2} & \underline{4}\\
4\,0 & 3\,3\,0\\
\underline{7} & \underline{3\,2\,9}\\
4\,7 & \quad 1\,0\,8\,0\,5\\
\underline{7} & \quad \underline{1\,0\,8\,0\,4}\\
5\,4\,0\,0 & \qquad\quad 1\\
\underline{2} & \\
5\,4\,0\,2 &
\end{array}$$

(*c*)

9	$\sqrt{90.00'00'00} = 9.487-$. Ans.
9	81
180	900
4	736
184	16400
4	15104
1880	1896) 1296.00 (.68+ or .7−
8	11376
1888	15840
8	14168
1896	

Having found the first three figures, we find the fourth by division, as shown.

(*d*) $\sqrt{.09} = .3$. Ans.

(57) (*a*)

6	36	$\sqrt[3]{.327'680'000} = .6894+$. Ans.
6	72	216
12	10800	111680
6	1504	98432
180	12304	13248000
8	1568	12650769
188	1387200	1424163) 5972310.0 (.41+ or .4+
8	18441	5696652
196	1405641	2756580
8	18522	1424163
2040	1424163	
9		
2049		
9		
2058		
9		
2067		

Here we find the first three figures in the regular way, and the fourth figure by the short method. See Art. **103.**

Explanation.—(1) When extracting the cube root, we divide the power into periods of three figures each. Always begin at the decimal point and proceed to the left in pointing off the whole number, and to the right in pointing off the decimal. In this power, $\sqrt[3]{.32768}$, a cipher must be annexed to 68 to complete the second decimal period. Cipher periods may now be annexed until the root has as many figures as desired.

(2) We find by trial that the largest number whose cube is contained in the first period, 327, is 6. Write 6 as the first figure of the root, also at the extreme left at the head of column (1). Multiply the 6 in column (1) by the first figure of the root, 6, and write the product, 36, at the head of column (2). Multiply the number in column (2) by the first figure of the root, 6, and write the product, 216, under the figures in the first period. Subtract and bring down the next period, 680, annexing it to the remainder, 111, thereby obtaining 111,680 for a new dividend. Add the first figure of the root, 6, to the number in column (1), obtaining 12, which we call the first correction; multiply the first correction, 12, by the first figure of the root, and we obtain 72 as the product, which added to 36 of column (2) gives 108. Annexing two ciphers to 108, we have 10,800 for the trial divisor. Dividing the dividend by the trial divisor, we see that it is contained about 8 times, so we write 8 as the second figure of the root. Add the first figure of the root to the first correction, and we obtain 18 as the second correction. To this annex one cipher, and add the second figure of the root, and we obtain 188. This multiplied by the second figure of the root, 8, equals 1,504, which added to the trial divisor, 10,800, forms the complete divisor, 12,304. Multiplying the complete divisor, 12,304, by 8, the second figure of the root, the result is 98,432. Write 98,432 under the dividend, 111,680; subtract, and there is a remainder of 13,248. To this remainder annex the next period, 000, thereby obtaining 13,248,000 for the next new dividend.

(3) Adding the second figure of the root, 8, to the number in column (1), 188, we have 196 for the first new correction. This multiplied by the second figure of the root, 8, gives 1,568. Adding this product to the last complete divisor, and

annexing two ciphers, gives 1,387,200 for the next trial divisor. Adding the second figure of the root, 8, to the first new correction, 196, we obtain 204 for the new second correction. Dividing the dividend by the trial divisor, 1,387,200, we see that it is contained about 9 times. Write 9 as the third figure of the root. Annex one cipher to the new second correction, and to this add the third figure of the root, 9, thereby obtaining 2,049. This multiplied by 9, the third figure of the root, equals 18,441, which added to the trial divisor, 1,387,200, forms the complete divisor, 1,405,641. Multiplying the complete divisor by the third figure of the root, 9, and subtracting, we have a remainder of 597,231. We then find the fourth figure by division, as shown.

```
(b)      4        1 6         ∛7 4'0 8 8 = 42.   Ans.
         4        3 2          6 4
         8        4 8 0 0      1 0 0 8 8
         4          2 4 4      1 0 0 8 8
     1 2 0        5 0 4 4
         2
     1 2 2
```

```
(c)     1 6                 ∛9 2'4 1 6 = 45.212–.   Ans.
  4     3 2                  6 4
  4     4 8 0 0              2 8 4 1 6
  8       6 2 5              2 7 1 2 5
  4     5 4 2 5                1 2 9 1 0 0 0
1 2 0     6 5 0                1 2 2 0 4 0 8
    5   6 0 7 5 0 0     612912 ) 7 0 5 9 2.0 0 0 ( .115
1 2 5       2 7 0 4              6 1 2 9 1 2
    5   6 1 0 2 0 4                9 3 0 0 8 0
1 3 0       2 7 0 8                6 1 2 9 1 2
    5   6 1 2 9 1 2                3 1 7 1 6 8 0
1 3 5 0                            3 0 6 4 5 6 0
      2                              1 0 7 1 2 0
1 3 5 2
      2
1 3 5 4
```

(*d*) 7	49	$\sqrt[3]{.373'248}$ = .72. Ans.
7	98	343
14	14700	30248
7	424	30248
210	15124	
2		
212		

(58)

1	1	$\sqrt[3]{2.000'000'000}$ = 1.259921+. Ans.
1	2	1
2	300	1000
1	64	728
30	364	272000
2	68	225125
32	43200	46875000
2	1825	42491979
34	45025	4755243) 4383021.000 (.9217 or .922–
2	1850	42797187
360	4687500	10330230
5	33831	9510486
365	4721331	8197440
5	33912	4755243
370	4755243	34421970
5		
3750		
9		
3759		
9		
3768		

This example shows what a great saving of figures is effected by using the short method. The figures obtained by the division are 9,217, thus making the last figures of the answer 922, according to Art. **91.** This is not correct in this case, the true answer, to eight decimal places, being 1.25992104+; hence, the first three figures found by division should be used in this case. The reason for the apparent failure of the method, in this case, to give the seventh figure of the root correctly, is because the fifth figure (the first obtained by division) is 9. Whenever the first figure obtained by division is 8 or 9, it is better to carry the root process one place further, before applying Art. **91,** if it is desired to obtain absolutely correct results.

(59) (*a*)

```
 1       1          ∛1'7 5 8.4 1 6'7 4 3 = 12.07.   Ans.
 1       2           1
 -       ---        -----
 2       3 0 0        7 5 8
 1         6 4       7 2 8
 ---     -----      ----------------
 3 0     3 6 4         3 0 4 1 6 7 4 3
   2       6 8         3 0 4 1 6 7 4 3
 ---     -------------  ----------------
 3 2     4 3 2 0 0 0 0
   2         2 5 2 4 9
 ---     -------------
 3 4     4 3 4 5 2 4 9
   2
 -------
 3 6 0 0
       7
 -------
 3 6 0 7
```

(*b*)

```
 1        1              ∛1'1 9 1'0 1 6 = 106.   Ans.
 1        2               1
 -        ---------      ---------------
 2        3 0 0 0 0         1 9 1 0 1 6
 1          1 8 3 6         1 9 1 0 1 6
 -----    ---------      ---------------
 3 0 0    3 1 8 3 6
     6
 -----
 3 0 6
```

(*c*) $\sqrt[3]{\frac{4}{32}} = \sqrt[3]{\frac{1}{8}} = \frac{\sqrt[3]{1}}{\sqrt[3]{8}} = \frac{1}{2}$. Ans.

(*d*) $\sqrt[3]{\frac{27}{512}} = \frac{\sqrt[3]{27}}{\sqrt[3]{512}} = \frac{3}{8}$. Ans.

(60)

1
1
2
1
30
4
34
4
38
4
420
4
424
4
428
4
4320
2
4322
2
4324

1
2
300
136
436
152
58800
1696
60496
1712
6220800
8644
6229444
8648
6238092

$\sqrt[3]{3.000'000'000} = 1.442250-$. Ans.
1
2000
1744
256000
241984
14016000
12458888
6238092) 1557112.000 (.2496 or .250—
12476184
30949360
24952368
59969920
56142828
3827092

(61) (a)

1
1
20
1
21
1
220
1
221

$\sqrt{1'23.21} = 11.1$. Ans.
1
23
21
221
221

(b)

1
1
200
7
207
7
2140
2
2142

$\sqrt{1'14.92'10} = 10.72+$. Ans.
1
1492
1449
4310
4284
26

(c)

7
7
140
0
1400
9
1409

$\sqrt{50'26'81} = 709$. Ans.
49
12681
12681

(d)

2
2
400
3
403

$\sqrt{.00'04'12'09} = .0203$. Ans.
00
04
4
1209
1209

(62) (*a*)

1	1	$\sqrt[3]{.006'500'000}$ = .18663—. Ans.
1	2	1
2	300	5500
1	304	4832
30	604	668000
8	368	602856
38	97200	103788) 65144.00 (.627 or .63—
8	3276	622728
46	100476	287120
8	3312	207576
540	103788	79544
6		
546		
6		
552		

(*b*)

2	4	$\sqrt[3]{.021'000'000}$ = .2759—. Ans.
2	8	8
4	1200	13000
2	469	11683
60	1669	1317000
7	518	1113875
67	218700	226875) 203125.0 (.89 or .9—
7	4075	1815000
74	222775	216250
7	4100	
810	226875	
5		
815		
5		
820		

(*c*)

2
2
4
2
6000
3
6003

4
8
12000000
18009
12018009

$\sqrt[3]{8'036'054'027} = 2,003$. Ans.
8
036054027
36054027

(*d*)

1
1
2
1
30
6
36

1
2
300
216
516

$\sqrt[3]{.000'004'096} = .016$. Ans.
000
004
1
3096
3096

(*e*)

2
2
4
2
60
5
65
5
70
5
750
7
757
7
764

4
8
1200
325
1525
350
187500
5299
192799
5348
198147

$\sqrt[3]{17.000'000} = 2.5713-$. Ans.
8
9000
7625
1375000
1349593
198147) 25407.00 (.128 or .13–
198147
559230
396294
162936

(63) (*a*) In this example, the index is 4, and equals 2 × 2. The root indicated is the fourth root, hence the square root must be extracted twice. Thus, $\sqrt[4]{\ }$ = $\sqrt{\ }$ of the $\sqrt{\ }$ and $\sqrt[4]{6,561} = \sqrt{\sqrt{6,561}} = \sqrt{81} = 9$. Ans.

$$\begin{array}{r} 8\\ \underline{8}\\ 160\\ \underline{1}\\ 161 \end{array} \qquad \begin{array}{r} \sqrt{6\,5'6\,1} = 81\\ \underline{6\,4}\qquad\quad\\ 1\,6\,1\qquad\\ \underline{1\,6\,1}\qquad \end{array} \qquad \sqrt{81} = 9. \text{ Ans.}$$

(*b*) In this example the index is 6, and 6 equals 3 × 2 or 2 × 3. The root indicated is the sixth root; hence, extract both the square and the cube roots, it making no particular difference as to which root is extracted first. Thus, $\sqrt[6]{\ }$ = $\sqrt[3]{\ }$ of the $\sqrt{\ }$, or $\sqrt{\ }$ of the $\sqrt[3]{\ }$.

Hence, $\sqrt[6]{117,649} = \sqrt[3]{\sqrt{117,649}} = \sqrt[3]{343} = 7$. Ans.

$$\begin{array}{r} 3\\ \underline{3}\\ 60\\ \underline{4}\\ 64\\ \underline{4}\\ 680\\ \underline{3}\\ 683 \end{array} \qquad \begin{array}{r} \sqrt{1\,1'7\,6'4\,9} = 343\\ \underline{9}\qquad\qquad\quad\\ 2\,7\,6\qquad\quad\\ \underline{2\,5\,6}\qquad\quad\\ 2\,0\,4\,9\qquad\\ \underline{2\,0\,4\,9}\qquad \end{array} \qquad \sqrt[3]{3\,4\,3} = 7. \text{ Ans.}$$

(*c*) $\sqrt[6]{.000064} = \sqrt[3]{\sqrt{.000064}} = .2$. Ans.

$\sqrt{.000064} = .008$. $\sqrt[3]{.008} = .2$. Hence, $\sqrt[6]{.000064} = .2$. Ans.

(*d*) $\sqrt[3]{\frac{3}{8}} = ?$ $\frac{3}{8} = .375$, since $8)\underline{3.000}$
.375

7	49	$\sqrt[3]{.375'000'000} = .72112+$.
7	98	343 Ans.
14	14700	32000
7	424	30248
210	15124	1752000
2	428	1557361
212	1555200	1559523) 194639.00 (.124 or .12+
2	2161	1559523
214	1557361	3868670
2	2162	3119046
2160	1559523	749624
1		
2161		
1		
2162		

Hence, $\sqrt[3]{\frac{3}{8}} = .72112+$. Ans.

(64) (*a*) $\sqrt{\frac{1225}{5476}} = \frac{\sqrt{1,225}}{\sqrt{5,476}}$.

3	$\sqrt{1\,2'2\,5} = 35$
3	9
60	325
5	325
65	

7	$\sqrt{5\,4'7\,6} = 74$.
7	49
140	576
4	576
144	

Hence, $\sqrt{\frac{1225}{5476}} = \frac{35}{74}$. Ans.

(*b*)

5 $\sqrt{.3\,3'6\,4} = .58.$ Ans.
5 2 5
1 0 0 8 6 4
8 8 6 4
1 0 8

(*c*)

3 $\sqrt{.1\,0'0\,0'0\,0'0\,0} = .31623-.$ Ans.
3 9
6 0 1 0 0
1 6 1
6 1 3 9 0 0
1 3 7 5 6
6 2 0 632) 1 4 4.0 0 (.227 or .23–
6 1 2 6 4
6 2 6 1 7 6 0
6 1 2 6 4
6 3 2 4 9 6

(*d*) $25.0\frac{3}{4} = 25.075.$

5 $\sqrt{2\,5.0\,7'5\,0'0\,0'0\,0'0\,0} = 5.00749+.$ Ans.
5 2 5
1 0 0 0 0 0 7 5 0 0 0
7 7 0 0 4 9
1 0 0 0 7 4 9 5 1 0 0
7 4 0 0 5 7 6
1 0 0 1 4 0 9 4 5 2 4 0 0
4 9 0 1 3 4 0 1
1 0 0 1 4 4 4 3 8 9 9 9
4
1 0 0 1 4 8 0
9
1 0 0 1 4 8 9

(*e*) $.000\frac{4}{9} = .0004444444+.$

2 $\sqrt{.0\,0'0\,4'4\,4'4\,4'4\,4} = .02108+.$ Ans.
2 0 0
4 0 0 4
1 4
4 1 4 4
1 4 1
4 2 0 0 3 4 4 4 4
8 3 3 6 6 4
4 2 0 8 7 8 0

(65) (*a*) $\sqrt[4]{2} = \sqrt{\sqrt{2}}$.

1	$\sqrt{2.00'00'00'00'00} = 1.41421356+$
1	1
20	100
4	96
24	400
4	281
280	11900
1	11296
281	60400
1	56564
2820	28284) 3836.0000(.13562 or .1356+
4	28284
2824	100760
4	84852
28280	159080
2	141420
28282	176600
2	169704
28284	6896

1	$\sqrt{1.41'42'13'56} = 1.1892+$. **Ans.**
1	1
20	41
1	21
21	2042
1	1824
220	21813
8	21321
228	49256
8	47564
2360	1692
9	
2369	
9	
23780	
2	
23782	

It is required in this problem to extract the fourth root of 2 to 4 decimal places; hence, we must extract the square root twice, since $\sqrt[4]{\ } = \sqrt{\ }$ of the $\sqrt{\ }$. In the first operation, we carry the root to 8 decimal places in order to carry the root in the second operation to 4 decimal places.

(*b*) $\sqrt[6]{6} = \sqrt[3]{\sqrt{6}}.$

```
2            √6.00'00'00'00'00'00 = 2.4494897428+
2             4
------        ----
40            200
 4            176
------        -----
44             2400
 4             1936
------         -----
480             46400
  4             44001
------          ------
484              239900
  4              195936
------           -------
4880              4396400
   9              3919104
------            -------
4889         489896) 477296.00000(.974280 or .97428+
   9                 4409064
------               -------
48980                 3638960
    4                 3429272
------                -------
48984                  2096880
    4                  1959584
------                 -------
489880                  1372960
     8                   979792
------                   -------
489888                   3931680
     8                   3919168
------                   -------
489896                     12512
```

It is required in this problem to find the sixth root of 6; hence, it is necessary to extract both the square and cube roots in succession, since the index, 6, equals 2×3 or 3×2. It makes no particular difference as to which root we extract first, but it will be more convenient to extract the square root first. The result has been carried to 10 decimal places; since the answer requires but 5 decimal places, the remaining decimals will not affect the cube root in the fifth decimal place, as the student can see for himself if he will continue the operation.

1
1
2
1
30
3
33
3
36
3
390
4
394
4
398
4
4020
8
4028
8
4036

1
2
300
99
399
108
50700
1576
52276
1592
5386800
32224
5419024
32288
5451312

$\sqrt[3]{2.449'489'742'800} = 1.34801-.$ Ans.
1
1449
1197
252489
209104
43385742
43352192
5451312) 33550.000 (.006 or .01–
32707872
842128

(66) (*a*)

1
1
20
7
27
7
340
7
347
7
354

$\sqrt{3.14'16} = 1.7725-.$ Ans.
1
214
189
2516
2429
354) 87.00 (.245+ or .25–
708
1620
1416
204

```
(b)       8            √.7 8'5 4'0 0 = .8862+.   Ans.
          8              6 4
       ----             -----
       1 6 0             1 4 5 4
          8              1 3 4 4
       ----             -------
       1 6 8               1 1 0 0 0
          8                1 0 5 9 6
       -------            ---------
       1 7 6 0        1772 )  4 0 4.0 ( .22 or .2+
             6                3 5 4 4
       -------                -------
       1 7 6 6                  4 9 6
             6
       -------
       1 7 7 2
```

```
(67)     (a)

1        1                 ∛3.1 4 1'6 0 0'0 0 0 = 1.4646−.
1        2                  1                          Ans.
-        -----              -----
2        3 0 0              2 1 4 1
1        1 3 6              1 7 4 4
--       -----              ---------
3 0      4 3 6                3 9 7 6 0 0
  4      1 5 2                3 6 8 1 3 6
--       ---------            -----------
3 4      5 8 8 0 0              2 9 4 6 4 0 0 0
  4        2 5 5 6              2 5 6 4 9 3 4 4
--       ---------              ---------------
3 8      6 1 3 5 6     6429888 ) 3 8 1 4 6 5 6.0 ( .59 or .6−
  4        2 5 9 2               3 2 1 4 9 4 4 0
-----    -------------           ---------------
4 2 0    6 3 9 4 8 0 0             5 9 9 7 1 2 0
    6        1 7 5 3 6
-----    -------------
4 2 6    6 4 1 2 3 3 6
    6        1 7 5 5 2
-----    -------------
4 3 2    6 4 2 9 8 8 8
    6
-------
4 3 8 0
      4
-------
4 3 8 4
      4
-------
4 3 8 8
```

(*b*)

$$\begin{array}{r}
8\\
\underline{8}\\
16\\
\underline{8}\\
2400\\
\underline{5}\\
2405\\
\underline{5}\\
2410
\end{array}
\qquad
\begin{array}{r}
64\\
\underline{128}\\
1920000\\
\underline{12025}\\
1932025\\
\underline{12050}\\
1944075
\end{array}
\qquad
\begin{array}{rl}
 & \sqrt[3]{.523'600'000} = .80599+ \text{ or } .8060-. \text{ Ans.}\\
 & \underline{512}\\
 & 11600000\\
 & \underline{9660125}\\
1944075) & 1939875.00\,(.99\\
 & \underline{17496675}\\
 & 19020750\\
 & \underline{17496675}
\end{array}$$

(68) $11.7 : 13 :: 20 : x.$

$11.7x = 13 \times 20$

$11.7x = 260$

$$x = \frac{260}{11.7}$$

The product of the means is equal to the product of the extremes.

$$\begin{array}{rl}
11.7\,) & 260.000\,(22.22+. \quad \text{Ans.}\\
 & \underline{234}\\
 & 260\\
 & \underline{234}\\
 & 260\\
 & \underline{234}\\
 & 260\\
 & \underline{234}\\
 & 26
\end{array}$$

(69) (*a*) $20+7 : 10+8 :: 3 : x.$

$27 : 18 :: 3 : x.$

$27x = 18 \times 3$

$27x = 54$

$x = \frac{54}{27} = 2.$ Ans.

(*b*) $(12)^2 : (100)^2 :: 4 : x.$

$144 : 10{,}000 :: 4 : x$

$144x = 10{,}000 \times 4$

$144x = 40{,}000$

$$x = \frac{40{,}000}{144}\Big)\,4\,0\,0\,0\,0.0\,(\,277.7+.\quad\text{Ans.}$$

$$\begin{array}{l}
2\,8\,8 \\ \hline
1\,1\,2\,0 \\
1\,0\,0\,8 \\ \hline
1\,1\,2\,0 \\
1\,0\,0\,8 \\ \hline
1\,1\,2\,0 \\
1\,0\,0\,8 \\ \hline
1\,1\,2
\end{array}$$

(70) (*a*) $\frac{4}{x} = \frac{7}{21}$ is equivalent to $4 : x :: 7 : 21$. The product of the means equals the product of the extremes. Hence,

$$\begin{aligned} 7x &= 4 \times 21 \\ 7x &= 84 \\ x &= \tfrac{84}{7} \text{ or } 12. \quad \text{Ans.} \end{aligned}$$

(*b*) In like manner,

$\frac{x}{24} = \frac{8}{16}$ is equivalent to $x : 24 :: 8 : 16$.

$$\begin{aligned} 16x &= 24 \times 8 \\ 16x &= 192 \\ x &= \tfrac{192}{16} = 12. \quad \text{Ans.} \end{aligned}$$

(*c*) $\frac{2}{10} = \frac{x}{100}$ is equivalent to $2 : 10 :: x : 100$.

$$\begin{aligned} 10x &= 2 \times 100 \\ 10x &= 200 \\ x &= \tfrac{200}{10} = 20. \quad \text{Ans.} \end{aligned}$$

(*d*) $\frac{15}{45} = \frac{60}{x}$ is equivalent to

$15 : 45 :: 60 : x$.

$$\begin{aligned} 15x &= 45 \times 60 \\ 15x &= 2{,}700 \\ x &= \frac{2{,}700}{15} = 180. \quad \text{Ans.} \end{aligned}$$

(*e*) $\frac{10}{150} = \frac{x}{600}$ is equivalent to

$10 : 150 :: x : 600$.

$$\begin{aligned} 150x &= 10 \times 600 \\ 150x &= 6{,}000 \\ x &= \frac{6{,}000}{150} = 40. \quad \text{Ans.} \end{aligned}$$

(71) $x : 5 :: 27 : 12.5.$

$$
\begin{array}{rl}
 & \underline{5} \\
12.5\,) & 135.0\ (\,10\tfrac{4}{5}.\quad \text{Ans.} \\
 & \underline{125} \\
 & \dfrac{100}{125} = \tfrac{4}{5}.
\end{array}
$$

(72) $45 : 60 :: x : 24.$

$$
\begin{aligned}
60x &= 45 \times 24 \\
60x &= 1{,}080 \\
x &= \frac{1{,}080}{60} = 18. \quad \text{Ans.}
\end{aligned}
$$

(73) $x : 35 :: 4 : 7.$

$$
\begin{aligned}
7x &= 35 \times 4 \\
7x &= 140 \\
x &= \tfrac{140}{7} = 20. \quad \text{Ans.}
\end{aligned}
$$

(74) $9 : x :: 6 : 24.$

$$
\begin{aligned}
6x &= 9 \times 24 \\
6x &= 216 \\
x &= \tfrac{216}{6} = 36. \quad \text{Ans.}
\end{aligned}
$$

(75)

$\sqrt[3]{1{,}000} : \sqrt[3]{1{,}331} :: 27 : x.$

$10 : 11 :: 27 : x.$

$$
\begin{aligned}
10x &= 297 \\
x &= \tfrac{297}{10} = 29.7. \\
&\qquad \text{Ans.}
\end{aligned}
$$

$$
\begin{array}{r}
1 \\
\underline{1} \\
2 \\
\underline{1} \\
30 \\
\underline{1} \\
31
\end{array}
$$

$\sqrt[3]{1{,}000} = 10.$

$\sqrt[3]{1{,}331} = 11.$

$$
\begin{array}{r}
1 \\
\underline{2} \\
300 \\
\underline{31} \\
331
\end{array}
\qquad
\begin{array}{r}
1'331\,(\,11 \\
\underline{1} \\
331 \\
\underline{331}
\end{array}
$$

(76) $64 : 81 :: 21^2 : x^2.$

Extracting the square root of each term of any proportion does not change its value, so we find that $\sqrt{64} : \sqrt{81} :: \sqrt{21^2} : \sqrt{x^2}$ is the same as

$$
\begin{aligned}
&8 : 9 :: 21 : x. \\
8x &= 189 \\
x &= 23.625. \quad \text{Ans.}
\end{aligned}
$$

(77) $7 + 8 : 7 :: 30 : x$ is equivalent to

$$
\begin{aligned}
&15 : 7 :: 30 : x. \\
15x &= 7 \times 30 \\
15x &= 210 \\
x &= \tfrac{210}{15} = 14. \quad \text{Ans.}
\end{aligned}
$$

(78) 2 ft. 5 in. = 29 in.; 2 ft. 7 in. = 31 in. Stating as a direct proportion, $29:31 = 2{,}480:x$. Now, it is easy to see that x will be greater than 2,480. But x should be less than 2,480, since when a man lengthens his steps, the number of steps required for the same distance is less; hence, the proportion is an inverse one, and

$$29:31 = x:2{,}480,$$

or $$31x = 71{,}920;$$

whence, $$x = 71{,}920 \div 31 = 2{,}320 \text{ steps. Ans.}$$

(79) This is evidently a direct proportion. 1 hr. 36 min. = 96 min.; 15 hr. = 900 min. Hence,

$$96:900 = 12:x,$$

or $$96x = 10{,}800;$$

whence, $$x = 10{,}800 \div 96 = 112.5 \text{ mi. Ans.}$$

(80) This is also a direct proportion; hence,

$$27.63:29.4 = .76:x,$$

or $$27.63x = 29.4 \times .76 = 22.344;$$

whence, $$x = 22.344 \div 27.63 = .808+ \text{ lb. Ans.}$$

(81) 2 gal. 3 qt. 1 pt. = 23 pt.; 5 gal. 3 qt. = 46 pt.

Hence, $$23:46 = 5:x,$$

or $$23x = 46 \times 5 = 230;$$

whence, $$x = 230 \div 23 = 10 \text{ days. Ans.}$$

(82) Stating as a direct proportion, and squaring the distances, as directed by the statement of the example, $6^2:12^2 = 24:x$. Inverting the second couplet, since this is an inverse proportion,

$$6^2:12^2 = x:24.$$

Dividing both terms of the first couplet (see Art. **129**) by 6,

$$1^2:2^2 = x:24; \text{ or, } 1:4 = x:24;$$

whence, $$4x = 24, \text{ or } x = 6 \text{ degrees. Ans.}$$

(83) Taking the dimensions as the causes,

$$\begin{array}{c|cl|l} \cancel{12} & 15 & & \\ \cancel{4} & 5 & & \\ 2 & \cancel{2} & = \cancel{12} & x, \text{ whence, } 2x = 75, \text{ or } x = \$37.50. \text{ Ans.} \\ \cancel{8} & \cancel{6} & & \end{array}$$

(84) 2 hr. = 120 min.; 14 hr. 28 min. = 868 min. Hence,

$$120:868 = 100:x,$$

or $$120x = 86{,}800;$$

whence, $$x = 723\tfrac{1}{3} \text{ gal. Ans.}$$

(85) Taking the dimensions as the causes,

$$\begin{array}{c|cc|l} 14 & \cancel{2} & & \\ \cancel{28} & \cancel{20} & & \\ \cancel{2} & & = \cancel{798} & x, \text{ whence, } 14x = 17\times 399 = 6{,}783, \\ \cancel{12} & 17 & 399 & \qquad \text{or } x = 484\tfrac{1}{2} \text{ bbl. Ans.} \\ \cancel{10} & \cancel{6} & & \end{array}$$

(86) 8 hr. 40 min. = 520 min. Hence,

$$444:1{,}060 = 520:x,$$

$$\text{or } x = \frac{1{,}060\times \overset{130}{\cancel{520}}}{\underset{111}{\cancel{444}}} = \frac{137{,}800}{111} = 1{,}241.44+ \text{ min.} = 20 \text{ hr. } 41.44+ \text{ min. Ans.}$$

(87) 1 min. = 60 sec. Hence,

$$5\tfrac{1}{2}:60 = 6{,}160:x,$$

or $$x = \frac{60\times 6{,}160}{5.5} = 67{,}200 \text{ ft. Ans.}$$

(88) Writing the statement as a direct proportion, $8:10 = 5:x$, it is easy to see that x will be greater than 5; but, it should be smaller, since by working longer hours, fewer men will be required to do the same work. Hence, the proportion is inverse. Inverting the second couplet,

$$8:10 = x:5,$$

or $$x = \frac{\overset{4}{\cancel{8}}\times\cancel{5}}{\underset{2}{\cancel{10}}} = 4 \text{ men. Ans.}$$

(89) Taking the times as the causes,

$$\begin{array}{r|lr|l} \cancel{20} & \cancel{25} & & 14 \\ & \cancel{5} & & \cancel{70} \\ & & = \cancel{540} & \cancel{630}; \\ \cancel{10} & x & \cancel{27} & \\ 2 & & 3 & \end{array}$$

whence, $3x = 2\times 14 = 28$, or $x = 9\frac{1}{3}$ hr. Ans.

(90) Taking the horsepowers as the effects, we have for the known causes in example 4, Art. **168,** 14^2, 500, and 48, and for the known effect 112 horsepower. Hence,

$$\begin{array}{r|l} 14^2 & 30^2 \\ 500 & 660 = 112 \\ 48 & 42 \end{array} \Bigg|\; x, \text{ or } \begin{array}{r|lr|l} & 9 & & \\ \cancel{14} & \cancel{900} & & \\ \cancel{196} & 22 & & \\ \cancel{5} & \cancel{110} & & \\ \cancel{500} & \cancel{660} & = \cancel{112} & x; \\ \cancel{6} & 3 & \cancel{8} & \\ \cancel{48} & \cancel{42} & & \end{array}$$

whence, $x = 9\times 22\times 3 = 594$ horsepower. Ans.

(91) First find the volume of the cylinder in cubic inches, as in the example, Art. **164.** The volume, multiplied by the weight of 1 cubic inch (.261 lb.) will evidently be the weight of the cylinder. Hence,

$$\begin{array}{r|l} 10^2 & 12^2 \\ 20 & 60 \end{array} = 1{,}570.8 \;\Big|\; x, \text{ or } \begin{array}{r|lr|l} 100 & 144 & & \\ & 3 & = 1{,}570.8 & x; \\ \cancel{20} & \cancel{60} & & \end{array}$$

$$\text{whence, } x = \frac{144\times 3\times 1{,}570.8}{100} = 1{,}771.11 \text{ lb. Ans.}$$

(92) Referring to the example in Art. **167,**

$$\begin{array}{r|l} 15 & 40 \\ 20^2 & 18^2 = 187 \\ 10 & 12 \end{array} \Bigg|\; x, \text{ or } \begin{array}{r|lr|l} 5 & \cancel{4} & & \\ \cancel{15} & \cancel{40} & & \\ 100 & & & \\ \cancel{400} & 324 & = 187 & x; \\ & 4 & & \\ \cancel{10} & \cancel{12} & & \end{array}$$

$$\text{whence, } x = \frac{324\times 4\times 187}{500} = 484.7 \text{ lb. Ans.}$$

ELEMENTARY ALGEBRA
AND
TRIGONOMETRIC FUNCTIONS.

---

(1) See Art. **99.**

(2) (*a*) $3x + 6 - 2x = 7x$. Transposing 6 to the second member, and $7x$ to the first member (Art. **143**),

$$3x - 2x - 7x = -6.$$

Combining like terms, $-6x = -6$;
whence, $x = 1$. Ans.

(*b*) $5x - (3x - 7) = 4x - (6x - 35)$.

Removing the parenthesis (Art. **50**),

$$5x - 3x + 7 = 4x - 6x + 35.$$

Transposing 7 to the second member, and $4x$ and $-6x$ to the first member,

$$5x - 3x - 4x + 6x = 35 - 7. \quad \text{(Art. } \mathbf{143.}\text{)}$$

Combining like terms, $4x = 28$;
whence, $x = 28 \div 4 = 7$. Ans.

(*c*) $(x + 5)^2 - (4 - x)^2 = 21x$.

Performing the operations indicated, the equation becomes

$$x^2 + 10x + 25 - 16 + 8x - x^2 = 21x.$$

Transposing, $x^2 - x^2 + 10x + 8x - 21x = 16 - 25$.
Combining like terms, $-3x = -9$.
Dividing by -3, $x = 3$.

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(3)

$$\begin{array}{l} a^3-a^2-2a-1)2a^6-4a^5-5a^4+3a^3+10a^2+7a+2(2a^3-2a^2-3a-2. \quad \text{Ans.} \\ \qquad\qquad\qquad\ \ \underline{2a^6-2a^5-4a^4-2a^3} \\ \qquad\qquad\qquad\qquad\ \ -2a^5-a^4+5a^3+10a^2 \\ \qquad\qquad\qquad\qquad\ \ \underline{-2a^5+2a^4+4a^3+2a^2} \\ \qquad\qquad\qquad\qquad\qquad\quad -3a^4+a^3+8a^2+7a \\ \qquad\qquad\qquad\qquad\qquad\quad \underline{-3a^4+3a^3+6a^2+3a} \\ \qquad\qquad\qquad\qquad\qquad\qquad\quad -2a^3+2a^2+4a+2 \\ \qquad\qquad\qquad\qquad\qquad\qquad\quad \underline{-2a^3+2a^2+4a+2} \end{array}$$

(4) (*a*)

$$\begin{array}{l} 2+4a-5a^2-6a^3 \\ \underline{7a^3\qquad\qquad\qquad\quad} \\ 14a^3+28a^4-35a^5-42a^6. \end{array}$$

Ans. (Art. **61.**)

(*b*)

$$\begin{array}{l} 4x^2-4y^2+6z^2 \\ \underline{3x^2y\qquad\qquad\quad} \\ 12x^4y-12x^2y^3+18x^2yz^2. \end{array}$$

Ans.

(*c*)

$$\begin{array}{l} 3b+5c-2d \\ \underline{6a\qquad\qquad\quad} \\ 18ab+30ac-12ad. \end{array}$$

Ans.

(5) Let x = number of miles he traveled per hour.

Then, $\frac{48}{x}$ = time it took him.

$\frac{48}{x+4}$ = time it would take him if he traveled 4 miles more per hour.

In the latter case the time would have been 6 hours; hence, the equation

$$\frac{48}{x+4} = 6.$$

Clearing of fractions, $48 = 6(x+4)$.
Dividing by 6, $8 = x+4$.
Transposing, $8-4 = x$.
$x = 4$. Ans.

(6) The square root of the fraction *a* plus *b* plus *c* over *n*, plus the square root of *a*, plus the fraction *b* plus *c* over *n*, plus the square root of the quantity *a* plus *b*, plus the fraction *c* over *n*, plus the parenthesis *a* plus *b*, times *c*, plus *a* plus *bc*.

(7) (*a*) Writing the work as follows, and canceling common factors in both numerator and denominator (Arts. **113** and **114**), we have

$$\frac{9m^2n^2}{8p^3q^3}\times\frac{5p^2q}{2xy}\times\frac{24x^2y^2}{90mn}$$

$$=\frac{9\times5\times24\times m^2\times n^2\times p^2\times q\times x^2\times y^2}{8\times2\times90\times m\times n\times p^3\times q^3\times x\times y}=\frac{3mnxy}{4pq^2}.\quad \text{Ans.}$$

(*b*) This problem may be written as follows, according to Art. **97**,

$$\frac{3ax+4}{1}\times\frac{a^2}{a(3ax+4)\,(3ax+4)}.$$

Canceling a and $(3ax+4)$, we have $\frac{a}{3ax+4}$. Ans.

(8) See Arts. **170** and **171.**

Sin 17° 28′ = .30015.
Sin 17° 27′ = .29987.

$30015-29987=28$, difference for 1′.
$28\times\frac{37}{60}=17$, difference for 37″.
$29987+17=30004$.

Hence, locating the decimal point,

Sin 17° 27′ 37″ = .30004.

Cos 17° 27′ = .95398.
Cos 17° 28′ = .95389.
$95398-95389=9$, difference for 1′.
$9\times\frac{37}{60}=6$, difference for 37″.
$95398-6=95392$.

Hence, Cos 17° 27′ 37″ = .95392.

Tan 17° 28′ = .31466.
Tan 17° 27′ = .31434.

$31466-31434=32$, difference for 1′.
$32\times\frac{37}{60}=20$, difference for 37″.
$31434+20=31454$.

Hence, Tan 17° 27′ 37″ = .31454.

$$\left.\begin{aligned}\text{Sin } 17^\circ\,27'\,37'' &= .30004.\\ \text{Cos } 17^\circ\,27'\,37'' &= .95392.\\ \text{Tan } 17^\circ\,27'\,37'' &= .31454.\end{aligned}\right\}\text{ Ans.}$$

(9) Let x = the capacity.

Then, $x - 42$ = amount held at first.

$$7(x-42) = x.$$
$$7x - 294 = x.$$
$$6x = 294.$$
x = 49 gallons. Ans.

(10) $\dfrac{c(a+b)+cd}{(a+b)c} = \dfrac{ac+bc+cd}{ac+bc}$. Canceling c, which is common to each term, we have $\dfrac{a+b+d}{a+b} = 1 + \dfrac{d}{a+b}$. Ans.

(11) The solution is exactly similar to that of example 8 preceding.

$$\left.\begin{array}{l}\text{Sin of } 63° \ 4' \ 51.8'' = .89165. \\ \text{Cos of } 63° \ 4' \ 51.8'' = .45274. \\ \text{Tan of } 63° \ 4' \ 51.8'' = 1.96949.\end{array}\right\} \text{Ans.}$$

(12) $(c^{-\frac{2}{5}})^{-\frac{1}{3}} = c^{\frac{2}{15}}$. Ans.

$$(m\sqrt{n^3})^{-\frac{1}{2}} = m^{-\frac{1}{2}}(n^{\frac{3}{2}})^{-\frac{1}{2}} = m^{-\frac{1}{2}}n^{-\frac{3}{4}} = \frac{1}{m^{\frac{1}{2}}n^{\frac{3}{4}}}. \quad \text{Ans.}$$

$(cd^{-2})^{\frac{1}{a}} = c^{\frac{1}{a}}d^{-\frac{2}{a}}$, or $\sqrt[a]{cd^{-2}}$, or $\sqrt[a]{\dfrac{c}{d^2}}$. Ans. (Art. **133.**)

(13) $\dfrac{3+2x}{2-x} - \dfrac{2-3x}{2+x} + \dfrac{16x-x^2}{x^2-4}$. If the denominator of the third fraction were written $4-x^2$, instead of x^2-4, the common denominator would then be $4-x^2$.

By Art. **99,** $\dfrac{16x-x^2}{x^2-4}$ becomes $-\dfrac{16x-x^2}{-x^2+4} = -\dfrac{16x-x^2}{4-x^2}$.

Hence, $\dfrac{3+2x}{2-x} - \dfrac{2-3x}{2+x} - \dfrac{16x-x^2}{4-x^2}$, when reduced to a common denominator, becomes

$$\frac{(3+2x)(2+x) - (2-3x)(2-x) - (16x-x^2)}{4-x^2}$$
$$= \frac{(6+7x+2x^2) - (4-8x+3x^2) - (16x-x^2)}{4-x^2}.$$

Removing the parentheses (Art. **50**), we have

$$\frac{6+7x+2x^2-4+8x-3x^2-16x+x^2}{4-x^2}.$$

Combining like terms in the numerator, we have

$$\frac{2-x}{4-x^2}.$$

Factoring the denominator (Art. **91**), we have

$$\frac{2-x}{(2+x)(2-x)}.$$

Canceling the common factor $(2-x)$, the result is equal to

$$\frac{1}{2+x}, \text{ or } \frac{1}{x+2}. \quad \text{Ans.} \quad (\text{Art. } \mathbf{24.})$$

(14) $$\frac{a^2+c^2+ac}{a^2+b^2-c^2-2ab} \times \frac{a^2+c^2-b^2-2ac}{a^3c+a^2c^2+ac^3}.$$

Arranging the terms, we have

$$\frac{a^2+ac+c^2}{a^2-2ab+b^2-c^2} \times \frac{a^2-2ac+c^2-b^2}{a^3c+a^2c^2+ac^3},$$

which, by the use of parentheses, becomes

$$\frac{a^2+ac+c^2}{(a^2-2ab+b^2)-c^2} \times \frac{(a^2-2ac+c^2)-b^2}{a^3c+a^2c^2+ac^3}.$$

By Art. **86,** we know that $a^2-2ab+b^2$ and $a^2-2ac+c^2$ are perfect squares, and may be written $(a-b)^2$ and $(a-c)^2$.

Factoring $a^3c+a^2c^2+ac^3$ by Case I, Art. **84,** we have

$$\frac{a^2+ac+c^2}{(a-b)^2-c^2} \times \frac{(a-c)^2-b^2}{ac(a^2+ac+c^2)}$$

$$= \frac{a^2+ac+c^2}{(a-b-c)(a-b+c)} \times \frac{(a-c-b)(a-c+b)}{ac(a^2+ac+c^2)}.$$

(Art. **91.**)

Canceling common factors and multiplying, we have

$$\frac{a-c+b}{(a-b+c)ac}, \text{ or } \frac{a+b-c}{ac(a-b+c)}. \quad \text{Ans.}$$

(15) If none of the terms is similar, the subtraction of one expression from another may be represented only by

connecting the subtrahend with the minuend by means of the sign $-$. Thus, if it is required to subtract $5a^3b-7a^2b^2+5ab^3$ from a^4-b^4, the result will be represented by $a^4-b^4-(5a^3b-7a^2b^2+5ab^3)$, which, on removing the parenthesis (Art. **50**), becomes $a^4-b^4-5a^3b+7a^2b^2-5ab^3$. From this result subtract $3a^4-4a^3b+6a^2b^2+5ab^3-3b^4$.

$$\begin{array}{rl}
a^4-\ b^4-5a^3b+7a^2b^2-\ 5ab^3 & \textit{minuend.} \\
\underline{-3a^4+3b^4+4a^3b-6a^2b^2-\ 5ab^3} & \textit{subtrahend}\text{, with signs changed.} \\
-2a^4+2b^4-\ a^3b+\ a^2b^2-10ab^3 & \textit{remainder.} \qquad \text{(Art. } \mathbf{47}.)
\end{array}$$

Or, $-2a^4-a^3b+a^2b^2-10ab^3+2b^4$. Ans. arranged according to the decreasing powers of a.

(16) $.27038 = \sin 15° \, 41' \, 12.9''$. (Art. **174.**)
$.27038 = \cos 74° \, 18' \, 47.1''$.
$2.27038 = \tan 66° \, 13' \, 43.2''$.

(17) (*a*) If the work done *on* a piston by the confined gases be considered positive, then the work done *by* the piston in pushing the gas out of the exhaust port may be considered negative.

(*b*) While in arithmetic we can add and subtract only positive quantities, in algebra we can perform these operations on both positive and negative quantities.

(18) (*a*) The value of a^0 is 1. (Art. **71.**)

(*b*) $\dfrac{a^0}{a^{-1}} = a$. Ans. (Art. **133.**)

(19) $-\dfrac{c-(a-b)}{c+(a+b)} = \dfrac{(a-b)-c}{c+(a+b)}$. Ans. (Art. **99.**)

(20) (*a*) By Art. **98,** the reciprocal of $\frac{24}{49} = 1 \div \frac{24}{49} = 1 \times \frac{49}{24} = \frac{49}{24}$. Ans.

(*b*) Since, by Art. **98,** a number may be found from its reciprocal by dividing 1 by the reciprocal, the number $= 1 \div 700 = .0014\frac{2}{7}$. Ans.

(21) (*a*) $\begin{array}{r} 3a-2b+3c \\ \underline{2a-8b-\ c} \end{array}$ becomes $\begin{array}{r} 3a-2b+3c \\ \underline{-2a+8b+\ c} \\ a+6b+4c \end{array}$

when the signs of the subtrahend are changed. Now, adding each term (with its sign changed) in the subtrahend to its corresponding term in the minuend, we have $(-2a) + (3a) = a$; $(+8b) + (-2b) = +6b$; $(+c) + (3c) = +4c$. Hence, $a + 6b + 4c$ equals the difference. Ans.

(*b*)

$$\begin{array}{l} 2x^3 - 3x^2y + 2xy^3 \\ \underline{x^3 \qquad\qquad\quad + y^3 - xy^2} \end{array} \quad \text{becomes} \quad \begin{array}{l} \quad 2x^3 - 3x^2y + 2xy^3 \\ \underline{-\ x^3 \qquad\qquad\qquad - y^3 + xy^2} \\ \quad x^3 - 3x^2y + 2xy^3 - y^3 + xy^2 \end{array}$$

when the signs of the subtrahend are changed. Adding each term in the subtrahend (with its sign changed) to its corresponding term in the minuend, we have $x^3 - 3x^2y + 2xy^3 - y^3 + xy^2$, which, arranged according to the decreasing powers of x, equals $x^3 - 3x^2y + xy^2 + 2xy^3 - y^3$. Ans.

(*c*)

$$\begin{array}{l} 14a + 4b - 6c - 3d \\ \underline{11a - 2b + 4c - 4d} \end{array}$$

On changing the sign of each term in the subtrahend, the problem becomes

$$\begin{array}{r} 14a + 4b - \ 6c - 3d \\ \underline{-11a + 2b - \ 4c + 4d} \\ 3a + 6b - 10c + \ d \end{array}$$

Adding each term of the subtrahend (with the sign changed) to its corresponding term in the minuend, the difference, or result, is $3a + 6b - 10c + d$. Ans.

(22) The numerical values of the following, when $a = 16$, $b = 10$, and $x = 5$, are:

(*a*) $(ab^2x + 2abx)4a = (16 \times 10^2 \times 5 + 2 \times 16 \times 10 \times 5) \times 4 \times 16$. It must be remembered that when no sign is expressed between symbols or quantities, the sign of multiplication is understood.

$(16 \times 100 \times 5 + 2 \times 16 \times 10 \times 5) \times 64 = (8{,}000 + 1{,}600) \times 64 = 9{,}600 \times 64 = 614{,}400$. Ans.

(*b*)

$$2\sqrt{4a}-\frac{2bx}{a-b}+\frac{b-x}{x}$$

$$= 2\sqrt{64}-\frac{2\times 10\times 5}{16-10}+\frac{10-5}{5}$$

$$= 16-\tfrac{100}{6}+1 = \frac{96-100+6}{6} = \tfrac{2}{6} = \tfrac{1}{3}.\quad \text{Ans.}$$

(*c*) $(b-\sqrt{a})\ (x^3-b^2)\ (a^2-b^2) = (10-\sqrt{16})\times(5^3-10^2)$ $\times(16^2-10^2) = (10-4)\ (125-100)\ (256-100) = 6\times 25\times 156$ $= 23,400$. Ans.

(23) (*a*)

$$\begin{array}{r} 4xyz \\ -3xyz \\ -5xyz \\ 6xyz \\ -9xyz \\ \underline{3xyz} \\ -4xyz.\ \text{Ans.} \end{array}$$

The sum of the coefficients of the positive terms we find to be $+13$, since $(+3)+(+6)+(+4) = (+13)$.

When no sign is given before a quantity, the $+$ sign must always be understood. The sum of the coefficients of the negative terms we find to be -17, since $(-9)+(-5)+(-3) = (-17)$. Subtracting the *lesser* sum from the *greater* and prefixing the sign of the greater sum $(-)$ (Art. **39,** rule II), we have $(+13)+(-17) = -4$. Since the terms are all alike, we have only to annex the common symbols xyz to -4, thereby obtaining $-4xyz$ for the result, or sum.

(*b*)

$$\begin{array}{rrr} 3a^2+ & 2ab+ & 4b^2 \\ 5a^2- & 8ab+ & b^2 \\ -a^2+ & 5ab- & b^2 \\ 18a^2- & 20ab- & 19b^2 \\ \underline{14a^2-} & \underline{3ab+} & \underline{20b^2} \\ 39a^2- & 24ab+ & 5b^2.\ \text{Ans.} \end{array}$$

When adding polynomials, always place like terms under each other. (Art. **46.**)

The coefficient of a^2 in the result will be 39, since $(+14)+(+18)+(-1)+(+5)+(+3) = 39$. When the coefficient of a term is not written, 1 is always understood to be its coefficient. (Art. **10.**) The coefficient of ab will be -24, since $(-3)+(-20)+(+5)+(-8)+(+2) = -24$. The coefficient of b^2 will be $(+20)+(-19)+(-1)+(+1)+(+4) = +5$. Hence, the result, or sum, is $39a^2-24ab+5b^2$.

(*c*)

$$\begin{array}{rrrrrrl}
4mn & +3ab & -4c & & & & \\
+2mn & -4ab & & +3x & +3m^2 & -4p & \\
\hline
6mn & -\ ab & -4c & +3x & +3m^2 & -4p. & \text{Ans.}
\end{array}$$

(24) (*a*) See Art. **13.**

(*b*) In multiplication, coefficients are multiplied, and exponents are added. In division the coefficients of the dividend are divided by those of the divisor, and the exponents of the divisor are subtracted from those of the dividend. See rules of multiplication and division.

(*c*) See Art. **55.**

(25) Now the side $BC = \sqrt{\overline{AB}^2 - \overline{AC}^2}$,

or $BC = \sqrt{17.69^2 - 9.75^2} = \sqrt{217.8736} = 14$ ft. 9 in.

To find the angle BAC, we have

$$\cos BAC = \frac{AC}{AB}, \text{ or } \cos BAC = \frac{9.75}{17.69} = .55115. \text{ (Art. } \mathbf{163.}\text{)}$$

$$.55115 = \cos 56^\circ\ 33'\ 15''.$$

Angle $ABC = 90^\circ -$ angle BAC, or $90^\circ - 56^\circ\ 33'\ 15'' = 33^\circ\ 26'\ 45''$.

$$\left.\begin{array}{l}
\text{Side } BC = 14 \text{ ft. } 9 \text{ in.} \\
\text{Angle } BAC = 56^\circ\ 33'\ 15''. \\
\text{Angle } ABC = 33^\circ\ 26'\ 45''.
\end{array}\right\}\text{Ans.}$$

(26) (*a*)

$$\frac{9x+20}{36} = \frac{4(x-3)}{5x-4} + \frac{x}{4}.$$

When the denominators contain both simple and compound expressions, it is best to remove the simple expressions first, and then remove each compound expression in order. Then, after each multiplication, the result should be reduced to the simplest form.

Multiplying both sides by 36,

$$9x + 20 = \frac{144(x-3)}{5x-4} + 9x,$$

or

$$\frac{144x - 432}{5x-4} = 20.$$

Clearing of fractions,

$$144x-432 = 100x-80.$$

Transposing and combining,

$$44x = 352;$$

whence, $x = 8.$ Ans.

(*b*) $ax-\frac{3a-bx}{2} = \frac{1}{2}$ becomes, when cleared of fractions,

$$2ax-3a+bx = 1.$$

Transposing and uniting terms,

$$2ax+bx = 3a+1.$$

Factoring, $(2a+b)x = 3a+1;$

whence, $x = \frac{3a+1}{2a+b}.$ Ans.

(*c*) $am-b-\frac{ax}{b}+\frac{x}{m} = 0$ becomes, when cleared of fractions,

$$abm^2-b^2m-amx+bx = 0.$$

Transposing, $bx-amx = b^2m-abm^2.$

Factoring, $(b-am)x = bm(b-am);$

whence, $x = \frac{bm(b-am)}{(b-am)} = bm.$ Ans.

(27) (*a*) *a* square *x* square, plus two *a* cube *b* fifth, minus the parenthesis *a* plus *b*.

(*b*) The cube root of *x*, plus *y* times the two-thirds power of the parenthesis *a* minus *n* square.

(*c*) The parenthesis *m* plus *n*, times the square of the parenthesis *m* minus *n*, times the parenthesis *m* minus the fraction *n* over two.

(28) (*a*) $16a^2b^3$; a^4+4ab; $4a^2-16a^3b+5a^6+7ax$.

(*b*) Since the terms are not alike, we can only indicate the sum, connecting the terms by their proper signs. (Art. **44.**)

(*c*) Multiplication: $4ac^2d$ means $4\times a\times c^2\times d$. (Art. **9.**)

(29) (*a*) $45x^7y^{10}-90x^5y^7-360x^4y^8$
$= 45x^4y^7(x^3y^3-2x-8y)$. Ans. (Art. **84.**)

(*b*) $a^2b^2+2abcd+c^2d^2 = (ab+cd)^2$. Ans. (Art. **87.**)

(*c*) $(a+b)^2-(c-d)^2 = (a+b+c-d)\ (a+b-c+d)$. Ans. (Art. **91.**)

(30) (*a*) On removing the vinculum, we have $2a-\{3b+[4c-4a-(2a+2b)]+[3a-b-c]\}$. (Art. **50.**)

Removing the parenthesis,

$$2a-\{3b+[4c-4a-2a-2b]+[3a-b-c]\}.$$

Removing the brackets,

$$2a-\{3b+4c-4a-2a-2b+3a-b-c\}.$$

Removing the brace,

$$2a-3b-4c+4a+2a+2b-3a+b+c.$$

Combining like terms, the result is $5a-3c$. Ans.

(*b*) Removing the parenthesis, we have

$$7a-\{3a-[2a-5a+4a]\}.$$

Removing the brackets,

$$7a-\{3a-2a+5a-4a\}.$$

Removing the brace,

$$7a-3a+2a-5a+4a.$$

Combining terms, the result is $5a$. Ans.

(*c*) Removing the parenthesis, we have

$$a-\{2b+[3c-3a-a-b]+[2a-b-c]\}.$$

Removing the brackets,

$$a-\{2b+3c-3a-a-b+2a-b-c\}.$$

Removing the brace,

$$a-2b-3c+3a+a+b-2a+b+c.$$

Combining like terms, the result is $3a-2c$. Ans.

(31) The hypotenuse $AB = \sqrt{\overline{AC}^2+\overline{BC}^2}$, or $AB = \sqrt{17.5^2+21.3^2} = \sqrt{759.94} = 27.57$, nearly. Ans.

$$\text{Tan } B = \frac{AC}{BC} = \frac{17.5}{21.3} = .82160. \quad \text{(Art. } \mathbf{164.}\text{)}$$

$$.82160 = \tan 39^\circ\ 24'\ 23''.$$

Angle $B = 39^\circ\ 24'\ 23''$. Ans.

Angle $A = 90^\circ - \text{angle } B = 90^\circ - 39^\circ\ 24'\ 23'' = 50^\circ\ 35'\ 37''$. Ans.

(32) $$t = \frac{W_1 s_1 t_1 + W_2 s_2 t_2}{W_1 s_1 + W_2 s_2}.$$

In order to transform this formula so that t_2 may stand alone in the first member, we must first clear of fractions. Performing this operation, we have

$$t W_1 s_1 + t W_2 s_2 = W_1 s_1 t_1 + W_2 s_2 t_2.$$

Transposing, we have

$$-W_2 s_2 t_2 = W_1 s_1 t_1 - t W_1 s_1 - t W_2 s_2.$$

Factoring, we have

$$-W_2 s_2 t_2 = W_1 s_1 t_1 - (W_1 s_1 + W_2 s_2) t;$$

whence, $$t_2 = \frac{(W_1 s_1 + W_2 s_2) t - W_1 s_1 t_1}{W_2 s_2}. \quad \text{Ans.}$$

(33) Let x = the part of the work which they all can do in 1 day when working together.

Then, since $\frac{1}{7\frac{1}{2}} = \frac{2}{15}$, $\quad \frac{1}{5} + \frac{1}{6} + \frac{2}{15} = x$;

or, clearing of fractions and adding,

$$15 = 30x, \text{ and } x = \tfrac{1}{2}.$$

Since they can do $\frac{1}{2}$ the work in 1 day, they can do all the work in 2 days. Ans.

(34) (*a*) $$\frac{\dfrac{1}{1-x} - \dfrac{1}{1+x}}{\dfrac{1}{1-x} + \dfrac{1}{1+x}} = \frac{\dfrac{1+x-1+x}{1-x^2}}{\dfrac{1+x+1-x}{1-x^2}}$$

$$= \frac{2x}{1-x^2} \div \frac{2}{1-x^2} = \frac{2x}{1-x^2} \times \frac{1-x^2}{2} = x. \quad \text{Ans.}$$

(Art. **127.**)

$$\frac{\dfrac{a^2}{b^3} + \dfrac{1}{a}}{\dfrac{a}{b^2} - \dfrac{a-b}{ab}} = \frac{\dfrac{a^3+b^3}{ab^3}}{\dfrac{a^2b - b^2(a-b)}{ab^3}} = \frac{\dfrac{a^3+b^3}{ab^3}}{\dfrac{a^2b - ab^2 + b^3}{ab^3}}$$

$$= \frac{a^3+b^3}{ab^3} \div \frac{a^2b - ab^2 + b^3}{ab^3} = \frac{a^3+b^3}{ab^3} \times \frac{ab^3}{b(a^2 - ab + b^2)}$$

$$= \frac{a+b}{b}. \quad \text{Ans.}$$

(*c*) $$\frac{1}{x+\cfrac{1}{1+\cfrac{x+1}{3-x}}}=\frac{1}{x+\cfrac{3-x}{4}}=\frac{4}{3x+3}.\quad \text{Ans.}$$

(Art. **127.**)

(35) (*a*) $6a^4b^4+a^3b^2-7a^2b^3+2abc+3$.

(*b*) $3+2abc+a^3b^2-7a^2b^3+6a^4b^4$.

(*c*) $1+ax+a^2+2a^3$. Written like this, the a in the second term is understood as having 1 for an exponent; hence, if we represent the first term by a^0, in value it will be equal to 1, since $a^0=1$. Therefore, 1 should be written as the first term when arranged according to the increasing powers of a.

(36) (*a*) According to Art. **131,** $x^{\frac{3}{4}}$ expressed radically is $\sqrt[4]{x^3}$;

$3x^{\frac{1}{2}}y^{-\frac{3}{2}}$ expressed radically is $3\sqrt{xy^{-3}}$;

$3x^{\frac{1}{6}}y^{-\frac{5}{6}}z^{\frac{1}{3}}=3\sqrt[6]{xy^{-5}z^2}$, since $z^{\frac{1}{3}}=z^{\frac{2}{6}}$. Ans.

(*b*) (See Art. **133.**) $$a^{-1}b^{\frac{1}{2}}+\frac{c^{-2}}{a+b}+(m-n)^{-1}-\frac{a^2b^{-2}c}{c^{-3}}$$

$$=\frac{b^{\frac{1}{2}}}{a}+\frac{1}{c^2(a+b)}+\frac{1}{m-n}-\frac{a^2c^4}{b^2}.\quad \text{Ans.}$$

(*c*) $\sqrt[7]{x^6}=x^{\frac{6}{7}}$. Ans. $\sqrt[3]{x^{-4}}=x^{-\frac{4}{3}}$. Ans.

$(\sqrt[4]{b^5x^{-2}})^3=(b^{\frac{5}{4}}x^{-\frac{1}{2}})^3=b^{\frac{15}{4}}x^{-\frac{3}{2}}$. Ans.

(37) (*a*) $$\frac{2ax+x^2}{a^3-x^3}\div\frac{x}{a-x}=\frac{x(2a+x)}{a^3-x^3}\times\frac{a-x}{x}.$$

(Art. **117.**)

Canceling common factors, the result is $\dfrac{2a+x}{a^2+ax+x^2}$. Ans.

$$\begin{array}{l}
a-x\,)\,a^3-x^3\,(\,a^2+ax+x^2\\
\qquad\;\;\underline{a^3-a^2x\qquad\quad}\\
\qquad\qquad\;\; a^2x-x^3\\
\qquad\qquad\;\;\underline{a^2x-ax^2\quad}\\
\qquad\qquad\qquad\;\; ax^2-x^3\\
\qquad\qquad\qquad\;\; ax^2-x^3
\end{array}$$

(*b*) Inverting the divisor and factoring, we have

$$\frac{3n(2m^2n-1)}{(2m^2n-1)(2m^2n-1)}\times\frac{(2m^2n+1)(2m^2n-1)}{3n}.$$

Canceling common factors, the result is $2m^2n+1$. Ans.

(*c*) $9+\frac{5y^2}{x^2-y^2}\div\left(3+\frac{5y}{x-y}\right)$ simplified gives

$\frac{9x^2-4y^2}{x^2-y^2}\div\frac{3x+2y}{x-y}$.

Inverting the divisor, we have $\frac{9x^2-4y^2}{x^2-y^2}\times\frac{x-y}{3x+2y}$. Canceling common factors, the result is $\frac{3x-2y}{x+y}$. Ans.

(38) (*a*)

$$\begin{array}{l}
2x^3+2x^2+2x-2\\
\underline{\quad x-1\qquad\qquad\qquad}\\
2x^4+2x^3+2x^2-2x\\
\underline{\qquad -2x^3-2x^2-2x+2}\\
2x^4 \qquad\qquad\qquad -4x+2.
\end{array}$$

Ans.

(*b*)

$$\begin{array}{l}
x^2-4ax+c\\
\underline{2x+a\qquad\qquad\qquad}\\
2x^3-8ax^2+2cx\\
\underline{\qquad ax^2 \qquad -4a^2x+ac}\\
2x^3-7ax^2+2cx-4a^2x+ac.
\end{array}$$

Ans.

(*c*)

$$\begin{array}{l}
-a^3+3a^2b-2b^3\\
\underline{\;5a^2+9ab\qquad\qquad\qquad\qquad}\\
-5a^5+15a^4b-10a^2b^3\\
\underline{\qquad -9a^4b \qquad\qquad +27a^3b^2-18ab^4}\\
-5a^5+6a^4b-10a^2b^3+27a^3b^2-18ab^4
\end{array}$$

Arranging the terms according to the decreasing powers of *a*, we have

$$-5a^5+6a^4b+27a^3b^2-10a^2b^3-18ab^4.$$

Ans.

(39) (*a*) $\frac{x}{x-y}+\frac{x-y}{y-x}$. If the denominator of the second fraction were written $x-y$, instead of $y-x$, then $x-y$ would be the common denominator.

By Art. **99,** the signs of the denominator and the sign before the fraction $\frac{x-y}{y-x}$ may be changed, giving $-\frac{x-y}{x-y}$. We now have

$$\frac{x}{x-y}-\frac{x-y}{x-y}=\frac{x-x+y}{x-y}=\frac{y}{x-y}. \quad \text{Ans.}$$

(*b*) $\frac{x^2}{x^2-1}+\frac{x}{x+1}-\frac{x}{1-x}$. If we write the denominator of the third fraction $x-1$ instead of $1-x$, x^2-1 will then be the common denominator.

By Art. **99,** the signs of the denominator and the sign before the fraction may be changed, thereby giving $\frac{x}{x-1}$. We now have

$$\frac{x^2}{x^2-1}+\frac{x}{x+1}+\frac{x}{x-1}=\frac{x^2+x(x-1)+x(x+1)}{x^2-1}$$

$$=\frac{x^2+x^2-x+x^2+x}{x^2-1}=\frac{3x^2}{x^2-1}. \quad \text{Ans.}$$

(*c*) $\frac{3a-4b}{7}-\frac{2a-b+c}{3}+\frac{13a-4c}{12}$ when reduced to a common denominator we have

$$\frac{12(3a-4b)-28(2a-b+c)+7(13a-4c)}{84}.$$

Expanding the terms and removing the parentheses, we have

$$\frac{36a-48b-56a+28b-28c+91a-28c}{84}.$$

Combining like terms in the numerator, we have as the result,

$$\frac{71a-20b-56c}{84}. \quad \text{Ans.}$$

(40) (*a*) Factoring each expression (Art. **87**), we have

$$9x^4+12x^2y^2+4y^4=(3x^2+2y^2)\,(3x^2+2y^2)=(3x^2+2y^2)^2.$$

Ans.

(*b*) $49a^4-154a^2b^2+121b^4$

$$=(7a^2-11b^2)\,(7a^2-11b^2)=(7a^2-11b^2)^2. \quad \text{Ans.}$$

(*c*) $64x^2y^2+64xy+16=16(2xy+1)^2$. Ans.

(41) (*a*) Arrange the dividend according to the decreasing powers of x and divide. Thus,

$$\begin{array}{l}
3x-1\,)\,9x^3+3x^2+x-1\,(\,3x^2+2x+1.\quad\text{Ans.}\\
\qquad\quad\underline{9x^3-3x^2\qquad}\\
\qquad\qquad\quad 6x^2+\;x\\
\qquad\qquad\quad\underline{6x^2-2x\qquad}\\
\qquad\qquad\qquad\quad 3x-1\\
\qquad\qquad\qquad\quad\underline{3x-1}
\end{array}$$

$$\begin{array}{l}
(b)\qquad a-b\,)\,a^3-2ab^2+b^3\,(\,a^2+ab-b^2.\quad\text{Ans.}\\
\qquad\qquad\quad\underline{a^3-a^2b\qquad}\\
\qquad\qquad\qquad\quad a^2b-2ab^2\\
\qquad\qquad\qquad\quad\underline{a^2b-\;ab^2\qquad}\\
\qquad\qquad\qquad\qquad -\;ab^2+b^3\\
\qquad\qquad\qquad\qquad \underline{-\;ab^2+b^3}
\end{array}$$

(*c*) Arranging the terms of the dividend according to th decreasing powers of x, we have

$$\begin{array}{l}
7x-3\,)\,7x^3-24x^2+58x-21\,(\,x^2-3x+7.\quad\text{Ans.}\\
\qquad\quad\underline{7x^3-\;3x^2\qquad}\\
\qquad\qquad -21x^2+58x\\
\qquad\qquad \underline{-21x^2+\;9x\qquad}\\
\qquad\qquad\qquad\quad 49x-21\\
\qquad\qquad\qquad\quad \underline{49x-21}
\end{array}$$

(42) See Arts. **3** and **4.**

(43) (*a*)

$$1+2x-\frac{4x-4}{5x}=\frac{5x+10x^2-4x+4}{5x}=\frac{10x^2+x+4}{5x}.\quad\text{Ans.}$$

(Art. **123.**)

(*b*) $\dfrac{3x^2+2x+1}{x+4}=3x-10+\dfrac{41}{x+4}.$ Ans. (Art. **122.**)

$$\begin{array}{l}
x+4\,)\,3x^2+\;2x+1\,(\,3x-10+\dfrac{41}{x+4}\\
\qquad\quad\underline{3x^2+12x\qquad}\\
\qquad\qquad -10x+\;1\\
\qquad\qquad \underline{-10x-40}\\
\qquad\qquad\qquad\quad 41
\end{array}$$

(44) The angle $B = 90° - \text{angle } A = 90° - 65° \; 13' \; 29'' = 24° \; 46' \; 31''$. Ans.

The side $BC = AB \times \sin A = 5.5 \text{ yd.} \times \sin 65° \; 13' \; 29'' = 5.5 \text{ yd.} \times .90796 = 4.9938 \text{ yd.}$ (Art. **177.**) 4.9938 yd. = 14 ft. $11\frac{3}{4}$ in. Ans.

Side $AC = AB \times \cos A = 5.5 \text{ yd.} \times .41906 = 2.3048 \text{ yd.}$ = 6 ft. 11 in., nearly. Ans.

(45) (*a*) $(x^4 - 1) \div (x^2 + 1) = (x^2 - 1)(x^2 + 1) \div (x^2 + 1) = x^2 - 1$. Ans. (Art. **91.**)

(*b*) $x^4 - 2x^2y^2 + y^4 = (x^2 - y^2)^2$. (Art. **87.**)

$(x^2 - y^2) = (x + y)(x - y)$. (Art. **91.**)

Then $(x^2 - y^2)^2 = (x^2 - y^2)(x + y)(x - y)$. Dividing this latter quantity by $(x - y)$ we have $(x^2 - y^2)(x + y)$. Ans.

Note that $x - y$ is a factor of $(x^2 - y^2)^2$ and hence of $x^4 - 2x^2y^2 + y^4$.

(46) (*a*) $$\frac{10x + 3}{3} - \frac{6x - 7}{2} = 10(x - 1).$$

Reducing the last member to a simpler form, the equation becomes

$$\frac{10x + 3}{3} - \frac{6x - 7}{2} = 10x - 10.$$

Clearing of fractions by multiplying each term of both members by 6, the common denominator, and changing the sign of each term of the numerator of the second fraction, since it is preceded by the minus sign, we have

$$20x + 6 - 18x + 21 = 60x - 60.$$

Transposing terms, $20x - 18x - 60x = -60 - 21 - 6$.

Combining like terms, $-58x = -87$.

Changing signs, $58x = 87$;

hence, $x = \frac{87}{58} = 1\frac{1}{2}$. Ans.

(*b*) $$(a^2 + x)^2 = x^2 + 4a^2 + a^4.$$

Performing the operation indicated in the first member, the equation becomes

$$a^4 + 2a^2x + x^2 = x^2 + 4a^2 + a^4.$$

Canceling a^4 and x^2 (Art. **144**),

$$2a^2x = 4a^2.$$

Dividing by $2a^2$, $x = 2$. Ans.

(*c*) $$\frac{x-1}{x-2} - \frac{x+1}{x+2} = \frac{3}{x^2-4}.$$

Clearing of fractions, the equation becomes

$$(x-1)(x+2) - (x+1)(x-2) = 3.$$

Expanding, $x^2 + x - 2 - x^2 + x + 2 = 3$.

Uniting terms, $2x = 3$.

$x = \frac{3}{2} = 1\frac{1}{2}$. Ans.

(47)
$$\begin{array}{r} 159^\circ\ 27'\ 34.6'' \\ 25^\circ\ 16'\ \ 8.7'' \\ 3^\circ\ 48'\ 53.0'' \\ \hline 188^\circ\ 32'\ 36.3'' \end{array}$$

(48) (*a*) $x + y + z - (x - y) - (y + z) - (-y)$ becomes $x + y + z - x + y - y - z + y$ on the removal of the parentheses. (Art. **50.**)

Combining like terms, $x - x + y + y - y + y + z - z = 2y$. Ans.

(*b*) $(2x - y + 4z) + (-x - y - 4z) - (3x - 2y - z)$ becomes $2x - y + 4z - x - y - 4z - 3x + 2y + z$ on the removal of the parentheses. (Arts. **50** and **51.**) Combining like terms,

$$2x - x - 3x - y - y + 2y + 4z - 4z + z = z - 2x. \text{ Ans.}$$

(*c*) $a - [2a + (3a - 4a)] - 5a - \{6a - [(7a + 8a) - 9a]\}$.

In this expression we find aggregation marks of different shapes, thus, [, (, and {. In such cases look for the corresponding part (whatever may intervene), and all that is included between the two parts of each aggregation mark must be treated as directed by the sign before it (Arts. **50** and **51**), no attention being given to any of the other aggregation marks. It is always best to begin with the *innermost* pair, and remove each pair of aggregation marks in order. First, removing the parentheses, we have

$$a - [2a + 3a - 4a] - 5a - \{6a - [7a + 8a - 9a]\}.$$

Removing the brackets, we have

$$a-2a-3a+4a-5a-\{6a-7a-8a+9a\}.$$

Removing the brace, we have

$$a-2a-3a+4a-5a-6a+7a+8a-9a.$$

Combining like terms, the result is $-5a$. Ans.

(49) (*a*) $$\begin{array}{r|l} -7my\,) & \underline{35m^3y+28m^2y^2-14my^3} \\ & -5m^2-\ \ 4my\ +\ 2y^2. \end{array}$$ Ans. (Art. **74.**)

(*b*) $$\begin{array}{r|l} a^4\,) & \underline{4a^4-3a^5b-a^6b^2} \\ & 4\ \ -3ab\ -a^2b^2. \end{array}$$ Ans.

(*c*) $$\begin{array}{r|l} 4x^2\,) & \underline{4x^3-8x^5+12x^7-16x^9} \\ & x\ -2x^3+\ 3x^5-\ 4x^7. \end{array}$$ Ans.

(50) Let x = the length of the post.

Then, $\frac{x}{5}$ = the part in the earth.

$\frac{3x}{7}$ = the part in the water.

From the conditions of the problem, we have, therefore, the following statement:

$$\frac{x}{5}+\frac{3x}{7}+13 = x;$$

from which $$7x+15x+455 = 35x;$$

$$-13x = -455;$$

and $$x = 35 \text{ feet.}$$ Ans.

(51) Let x = the whole quantity.

Then, $\frac{2x}{3}+10$ = the quantity of niter.

$\frac{x}{6}-4\frac{1}{2}$ = the quantity of sulphur.

$\frac{1}{7}\left(\frac{2x}{3}+10\right)-2$ = the quantity of charcoal.

Hence, $$x = \frac{2x}{3}+10+\frac{x}{6}-4\tfrac{1}{2}+\frac{1}{7}\left(\frac{2x}{3}+10\right)-2.$$

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Clearing of fractions and expanding terms,

$$42x = 28x + 420 + 7x - 189 + 4x + 60 - 84.$$

Transposing,

$$42x - 28x - 7x - 4x = 420 - 189 + 60 - 84.$$

$$3x = 207.$$

$$x = 69 \text{ lb., the quantity of gunpowder.}$$

Ans.

$$\frac{2x}{3} + 10 = \frac{2 \times 69}{3} + 10 = 56 \text{ lb., the quantity of niter.} \quad \text{Ans.}$$

$$\frac{x}{6} - 4\tfrac{1}{2} = \tfrac{69}{6} - 4\tfrac{1}{2} = 7 \text{ lb., the quantity of sulphur.} \quad \text{Ans.}$$

$$56 \text{ lb.} \times \tfrac{1}{7} - 2 \text{ lb.} = 6 \text{ lb., the quantity of charcoal.} \quad \text{Ans.}$$

(52) (*a*) See Art. **136.** The cube root of -125 is -5. Dividing each of the exponents of the literal part by 3, the index of the root, the cube root of $x^3y^6z^9$ is $x^{\frac{3}{3}}y^{\frac{6}{3}}z^{\frac{9}{3}} = xy^2z^3$; hence, $\sqrt[3]{-125x^3y^6z^9} = -5xy^2z^3$. Ans.

(*b*) and (*c*) Proceed exactly as in (*a*). $\sqrt[4]{10{,}000} = \pm 10$ and $\sqrt[5]{243} = 3$; $\sqrt[4]{a^{16}b^{20}c^8} = a^{\frac{16}{4}}b^{\frac{20}{4}}c^{\frac{8}{4}} = a^4b^5c^2$, and $\sqrt[5]{m^{15}n^{20}} = m^3n^4$.

Hence, $\sqrt[4]{10{,}000a^{16}b^{20}c^8} = \pm 10a^4b^5c^2$. Ans.

$$\sqrt[5]{243m^{15}n^{20}} = 3m^3n^4. \quad \text{Ans.}$$

(*d*) Dividing the exponent of each letter in both numerator and denominator by 5, the index of the root,

$$\sqrt[5]{-\frac{x^5y^{10}z^{15}}{a^{20}b^{15}c^{10}d^5}} = -\frac{xy^2z^3}{a^4b^3c^2d}. \quad \text{Ans.}$$

(53) Using the proportion of Art. **183,**

$$AB : BC = \sin C : \sin A,$$

or

$$70 : 42 = \sin C : \sin 36° \, 10'.$$

Hence, $\sin C = \frac{70}{42} \times \sin 36° \, 10' = \frac{70}{42} \times .59014 = .98357$.

The angle whose sine is .98357 is 79° 36′; hence angle $C = 79° \, 36'$. Ans.

Angle $B = 180° - (A + C) = 180° - (36° \, 10' + 79° \, 36') = 64° \, 14'$. Ans.

Using the proportion again,

$A\,C : B\,C = \sin B : \sin A$;

or $A\,C : 42 \text{ ft.} = \sin 64^\circ\ 14' : \sin 36^\circ\ 10' = .90057 : .59014$.

Hence, $A\,C = \dfrac{42 \text{ ft.} \times .90057}{.59014} = 64.1$ ft., nearly. Ans.

(54) (*a*) See Art. **159.**

$180^\circ - 72^\circ\ 11'\ 36'' = 107^\circ\ 48'\ 24''$. Ans.

(*b*) See Art. **158.**

$90^\circ - 22^\circ\ 34'\ 17'' = 67^\circ\ 25'\ 43''$. Ans.

(55) Angle $B = 180^\circ - (A + C)$

$= 180^\circ - (57^\circ\ 34.5' + 44^\circ\ 22.5') = 78^\circ\ 3'$. Ans.

$A\,C : A\,B = \sin B : \sin C = \sin 78^\circ\ 3' : \sin 44^\circ\ 22.5'$

$= .97833 : .69936$.

Hence, $A\,C = \dfrac{344 \text{ ft.} \times .97833}{.69936} = 481.22$ ft. Ans.

$B\,C : A\,B = \sin A : \sin C = \sin 57^\circ\ 34.5' : \sin 44^\circ\ 22.5'$

$= .8441 : .69936$.

Hence, $B\,C = \dfrac{344 \text{ ft.} \times .8441}{.69936} = 415.19$ ft. Ans.

PHYSICS.

(1) See Art. **32.** $127 \div 62.42 = 2.034$ = specific gravity. Ans.

(2) The current is flowing from the north towards the south.

(3) See Art. **33.** $\frac{9.823 \times 62.42}{1,728} = .3548$ lb. Ans.

(4) (*a*) and (*b*) See Art. **167.**

(5) (*a*) and (*b*) See Art. **1.**

(6) See Art. **213.**

(7) (*a*) See Art. **159.**

(*b*) See Art. **160.**

(8) At the bottom of the valley; see also Arts. **29** and **30.**

(9) Using formula **13,**

$$p_1 = p\left(\frac{460 + t_1}{460 + t}\right) = 12 \times \left(\frac{460 + 300}{460 + 60}\right) = 17.54 \text{ lb. per sq. in.}$$ Ans.

(10) See Art. **168.**

(11) (*a*) See Arts. **4** and **26.**

(*b*) See Art. **4.**

(12) The area of the surface of the sphere = $(\text{diameter})^2 \times 3.1416 = 4 \times 4 \times 3.1416 = 50.2656$ sq. in. From Art. **199,**

the electric density of a charged body is found by dividing the number of units of electricity by the area of the surface upon which it resides; therefore, the electric density of the sphere is $400 \div 50.2656 = 7.9577$ units of electricity per square inch. Ans.

(13) (*a*) See Art. **134.**

(*b*) See Art. **135.**

(14) (*a*) $7.14 \div .08 = 89.25$ cu. ft., the new volume. Ans.

(*b*) If 1 cu. ft. weighs .08 lb., 1 lb. contains $1 \div .08 = 12.5$ cu. ft. Hence, using formula **15,** $p\,V = .37052\,W\,T$, or $T = \frac{p\,V}{.37052\,W} = \frac{22.05 \times 89.25}{.37052 \times 7.14} = 743.887$ ($1\frac{1}{2}$ atmospheres $= 14.7 \times 1\frac{1}{2} = 22.05$ lb.). Hence, $743.887 - 460 = 283.887°$. Ans.

(*c*) Using formula **15,**

$$V = \frac{.37052\,W\,T}{p} = \frac{.37052 \times 7.14 \times (460 + 75)}{22.05} = 64.188 \text{ cu. ft.}$$ Ans.

(15) (*a*), (*b*), and (*c*) See Arts. **90** and **91.**

(16) By formula **27,**

$$t = \frac{W_1\,s_1\,t_1 + W_2\,s_2\,t_2 + W_3\,s_3\,t_3}{W_1\,s_1 + W_2\,s_2 + W_3\,s_3}$$

$$= \frac{2.25 \times .0314 \times 40 + 4 \times .65 + 1.75 \times .1298 \times 62}{2.25 \times .0314 + 4 + 1.75 \times .1298} = 64.43°.$$ Ans.

(17) See Art. **32.**

(18) See Art. **206.**

(19) Using formula **10,** $p\,W_1 = p_1\,W$,

$$3.5 \times 14.7 \times 2 = p_1 \times 13;$$

hence, $p_1 = \frac{14.7 \times 3.5 \times 2}{13} = 7.915+$ lb. per sq. in. Ans.

(20) See Art. **198.**

(21) (*a*) and (*b*) See Art. **3.**

(22) (*a*) and (*b*) See Art. **64.**

(23) Southward, as unlike poles attract each other; see also Art. **170.**

(24) Towards the east; see Art. **219.**

(25) See Art. **7.**

(26) (*a*) Using formula **2,** $wR = dW$,

$$W = \frac{w \cdot R}{d} = \frac{1 \times 4,000}{100} = 40 \text{ lb. Ans.}$$

(*b*) Using formula **3,** $wd^2 = WR^2$,

$$w = \frac{WR^2}{d^2} = \frac{40 \times 4,000^2}{4,100^2} = 38.073 \text{ lb. Ans.}$$

(27) (*a*) and (*b*) See Art. **182.**

(28) Using formula **3,** $wd^2 = WR^2$,

$$d = \sqrt{\frac{4,000^2 \times 2}{\frac{3}{16}}} = 13,064 \text{ mi., nearly.}$$

$$13,064 - 4,000 = 9,064 \text{ mi. Ans.}$$

(29) See Art. **211.**

(30) (*a*) $2,000 \div 4 = 500$ = weight of 1 cubic foot. $500 \div 62.5 = 8$ = specific gravity. Ans.

(*b*) $\frac{500}{1,728} = .2894$ lb. Ans.

(31) As soon as the two gilt balls come in contact, the $+4$ charge on ball *A* neutralizes *a* -4 charge on ball *B*, leaving a negative charge of -20 units, which immediately divides equally between the two balls. A charge of -10 units on each ball produces a repulsion of $10 \times 10 = 100$ dynes between the two balls at a distance of 1 centimeter; and at a distance of 2 centimeters the force will be $\frac{100}{2^2} = 25$ dynes, since the force exerted between them varies inversely as the square of the distance. (See Art. **190.**) Ans.

(32) See Art. **12.**

(33) See Art. **182.**

(34) (*a*) Using formula **5**,

$$\frac{w}{(W-W')-(W_1-W_2)}$$

$$=\frac{11.25}{(91.25-41)-(16\times5-3\tfrac{1}{8}\times16)}=.555.\quad\text{Ans.}$$

(*b*) Using formula **4**,

$$\text{Sp. Gr.}=\frac{W}{W-W'}=\frac{80}{80-50}=2.667.\quad\text{Ans.}$$

(35) Positive; see Arts. **187** and **198.** A negative charge is developed on sulphur when rubbed with silk (Art. **187**). This negative charge will induce a *bound* positive charge on the cover towards the sulphur, and a free negative charge on the opposite side of the cover, which will neutralize with the earth and leave the positive charge alone on the cover.

(36) 2 lb. = 32 oz. 32 − 10 = 22 oz. = loss of weight of the bottle in water. 32 + 16 = 48 oz. = weight of bottle and sugar in air. 48 − 16 = 32 oz. = loss of weight of bottle and sugar in water. 32 − 22 = 10 oz. = loss of weight of sugar in water = weight of a volume of water equal to the volume of the sugar. Then, by formula **4**,

$$\text{Sp. Gr.}=\frac{W}{W-W'}=\frac{16}{10}=1.6.\quad\text{Ans.}$$

(37) (*a*) Negative.

(*b*) Negative.

(*c*) Positive. See Art. **187.**

(38) (*a*) $W=$ 2 lb. $8\frac{1}{2}$ oz. = 40.5 oz.

$w=$ 12 oz.

$W'=$ 1 lb. 11 oz. = 27 oz.

By formula **6**,

$$\text{Sp. Gr.}=\frac{W-w}{W'-w}=\frac{40.5-12}{27-12}=1.9.\quad\text{Ans.}$$

(*b*) See Art. **44.** $\frac{28.5}{1.9\times.03617\times16}=25.919$ cu. in. Ans.

(39) See Art. **188.**

(40) See Art. **50.**

(41) Using formula **7,**

$$p_1 = \frac{p\,v}{v_1} = \frac{(14.7 \times 3) \times 1}{2.5} = 17.64 \text{ lb. Ans.}$$

(42) See Art. **189.**

(43) It means that the mercurial volume of the gauge stands 23 inches high, or in other words that there are 23 inches of vacuum. $\frac{30 - 23}{30} \times 14.7 = 3.43$ lb. per sq. in. = pressure in condenser. Ans. See also Art. **50.**

(44) (*a*) Since the force exerted between two statically charged bodies is equal to the product of their two respective charges (Art. **191**), and a unit quantity of electricity attracts another unit quantity of opposite sign with a force of 1 dyne at a distance of 1 centimeter, then the force of attraction exerted between the two gilt balls at a distance of 1 centimeter is $20 \times 5 = 100$ dynes. Ans.

(*b*) According to the law of attraction and repulsion of static charges (Art. **190**), the force exerted between them varies inversely as the square of the distance; consequently, if $f =$ the force at 5 centimeters, then

$$100 : f = 5^2 : 1, \text{ or } 5^2 f = 100, \text{ or } f = 4 \text{ dynes. Ans.}$$

(45) Using formula **25,**

$$p\,V = R\,W\,T; \text{ or } 18 \times 100 = 5.34946 \times (80 + 460) \times W.$$

$$W = \frac{18 \times 100}{5.34946 \times (80 + 460)} =$$

$$p\,V = R\,W\,T; \text{ or } 18 \times 100 = 5.34946 \times W \times (80 + 460).$$

$$W = \frac{18 \times 100}{5.34946 \times (80 + 460)} = .6231 \text{ lb. Ans.}$$

(46) Using formula **12,**

$$v_1 = v\left(\frac{460 + t_1}{460 + t}\right) = 4\left(\frac{460 + 115}{460 + 40}\right) = 4.6 \text{ cu. ft. Ans.}$$

(47) See Art. **179.**

(48) Substituting in formula **13,** $p = 40$, $t = 120$, and $t_1 = 55$,

$$p_1 = 40\left(\frac{460+55}{460+120}\right) = 35.517 \text{ lb. per sq. in. Ans.}$$

(49) See Art. **123.**

(50) See Art. **163.**

(51) Using formula **15,**

$$p\,V = .37052\,W\,T, \text{ or } W = \frac{p\,V}{.37052\,T}.$$

$T = 460° + 60° = 520°$; therefore,

$$W = \frac{14.7 \times 1}{.37052 \times 520} = .0763 \text{ lb. Ans.}$$

(52) See Art. **186.**

(53) For definition see Art. **80.**

(54) (*a*) Using formula **23,** $v = V C_3 t$, then

$$v = 4 \times 4 \times 20 \times .00002058 \times 1{,}200 = 7.903 \text{ cu. in. Ans.}$$

(*b*) Using formula **21,** $l = L\,C_1 t$, then

$$l = 20 \times .00000686 \times 1{,}200 = .16464 \text{ in. Ans.}$$

(*c*) Using formula **22,** $a = A\,C_2 t$, then

$$a = 4 \times 4 \times .00001372 \times 1{,}200 = .2634 \text{ sq. in. Ans.}$$

(55) For (*a*) and (*b*) see Art. **148.** For (*c*) see Art. **147.**

(56) See Art. **180.**

(57) Using formula **17,** $VP = p\,v + p_1 v_1$; therefore,

$$P \times 7.5 = 14.7 \times 2 \times 7.5 + 40 \times 7.5$$

or $$P = 69.4 \text{ lb. per sq. in. Ans.}$$

(58) Extending formula **17** to include three gases, we have $P\,V = p_1 v_1 + p_2 v_2 + p_3 v_3$, or $P \times 40 = 1 \times 12 + 2 \times 10 + 3 \times 8$. Hence, $P = \frac{56}{40} = 1.4$ atmos. $= 1.4 \times 14.7 = 20.58$ lb. per sq. in. Ans.

(59) By formula **21,** $l = L\,C_1 t$,

$$C_1 = .00000599,\ L = 900 \times 12,\ t = 90° - 28°.$$

Therefore,

$$l = 900 \times 12 \times .00000599 \times (90 - 28) = 4.01 \text{ in. Ans.}$$

(60) See Art. **178.**

(61) (*a*) The Fahrenheit thermometer is generally used in the United States and England.

(*b*) $R : C = 80 : 100$; therefore,

$$80^\circ : 100^\circ = 44^\circ : 55^\circ \text{ C. Ans.}$$

(*c*) Using formula **19,** $t_f = \frac{9}{5}t_c + 32$; substituting the proper values, we obtain $t_f = \frac{9}{5} \times 55 + 32 = 131^\circ$ F. Ans.

(62) See Art. **192** for (*a*) and (*b*).

(63) See Art. **124.**

(64) See Art. **145** and Fig. 24. In the latter, P is the refracting edge, a the refracting angle, and $S\,Q$ the base of the prism.

(65) North; see Art. **170.**

(66) According to formula **18,** $PV = \left(\frac{p_1 v_1}{T_1} + \frac{p_2 v_2}{T_2}\right)T$,

$$20 \times (18 + 13) = \left(\frac{14.7 \times 13}{533} + \frac{30 \times 18}{513}\right)T = 1.411168\ T.$$

Therefore, $T = \dfrac{20 \times 31}{1.411168} = 439.35^\circ$. Since this is less than 460°, the temperature is $460 - 439.35 = 20.65^\circ$ below zero, or -20.65°. Ans.

(67) Negative; see Art. **187.**

(68) Use formula **18,** $PV = \left(\frac{p_1 v_1}{T_1} + \frac{p_2 v_2}{T_2}\right)T$.

$T = 110^\circ + 460^\circ = 570^\circ$; $T_1 = 100^\circ + 460^\circ = 560^\circ$; $T_2 = 130^\circ + 460^\circ = 590^\circ$. Therefore,

$$V = \frac{\left(\frac{90 \times 40}{560} + \frac{80 \times 57}{590}\right)570}{120} = 67.248 \text{ cu. ft. Ans.}$$

(69) Use formula **20,** $t_c = (t_f - 32)\frac{5}{9}$.

(*a*) $t_c = (-10 - 32) \times \frac{5}{9} = -23\frac{1}{3}^\circ$ C. Ans.

(*b*) $t_c = (25 - 32) \times \frac{5}{9} = -3\frac{8}{9}^\circ$ C. Ans.

(*c*) $t_c = (2{,}200 - 32) \times \frac{5}{9} = 1{,}204\frac{4}{9}^\circ$ C. Ans.

(70) See Art. **173.**

(71) According to formula **12,** $v_1 = v\left(\frac{460+t_1}{460+t}\right)$;

or, $$v = 12\left(\frac{460+390}{460+65}\right) = 19.4286;$$

$$19.4286 - 12 = 7.4286 \text{ cu. ft.} \quad \text{Ans.}$$

(72) (*a*) See Art. **193.**

(*b*) See Art. **194.**

(73) 1 calorie = 3.96 B. T. U. (Art. **106**). Hence, (*a*) 798 B. T. U. = $798 \div 3.96$ = 201.515 calories. Ans.

(*b*) 40×3.96 = 158.4 B. T. U. Ans.

(74) Increase in diameter = 4.001 − 3.9985 = .0025″. C_1 = .00000599 from Table 2. Hence, using formula **21,** $l = L\,C_1\,t$ and $t = \frac{l}{L\,C_1} = \frac{.0025}{3.9985 \times .00000599} = 104.4°$, nearly. Then, 104.4 + 80 = 184.4°, nearly. Ans.

(75) Substitute in formula **26,** $W = 360$, $t_1 - t = 104.4$, and $s = .1165$; then, $n = W(t_1 - t)s = 360 \times 104.4 \times .1165$ = 4,378.536 B. T. U.

Since 12 per cent. of the total heat is lost by radiation, the above number of B. T. U. must be the number remaining after subtracting the heat units lost by radiation from the total number; or, it is 100 − 12 = 88 per cent. of the total number. Hence, dividing by .88, we obtain $4,378.536 \div .88$ = 4,975.6 B. T. U., total heat required. Ans.

(76) By using formula **27,**

$$t = \frac{W_1 s_1 t_1 + W_2 s_2 t_2 + W_3 s_3 t_3}{W_1 s_1 + W_2 s_2 + W_3 s_3};$$

or, $$t_1 = \frac{(W_1 s_1 + W_2 s_2 + W_3 s_3)t - W_2 s_2 t_2 - W_3 s_3 t_3}{W_1 s_1}$$

$$= \frac{(1 \times .1298 + 3.25 \times 1 + 1.5 \times .0951)128 - 3.25 \times 85 - 1.5 \times .0951 \times 85}{1 \times .1298}$$

$$= 1{,}252°. \quad \text{Ans.}$$

(77) Using formula **29,**

$$x = \frac{100\,z}{19.6\,y} = \frac{100 \times 29}{19.6 \times 12} = 12.33\%. \quad \text{Ans.}$$

(78) See Art. **179.**

(79) See Art. **107.**

(80) (*a*) $30-17.5 = 12.5$ in. = original tension of gas in inches of mercury. $30-5 = 25$ in. = new tension in inches of mercury. $VP = vp + v_1 p_1$ (formula **17**), or $6.7\times 25 = 6.7\times 12.5 + v_1\times 30$.

$$v_1 = \frac{6.7\times 25 - 6.7\times 12.5}{30} = 2.79\tfrac{1}{6} \text{ cu. ft.} \quad \text{Ans.}$$

(*b*) To produce a vacuum of 0 inches,

$$v_1 = \frac{6.7\times 30 - 6.7\times 12.5}{30} = 3.908 \text{ cu. ft.} \quad \text{Ans.}$$

(81) Applying formula **12,**

$$V_1 = V\left(\frac{460+t_1}{460+t}\right) = \frac{4,516}{1,728}\left(\frac{460+80}{460+260}\right) = 1.96 \text{ cu. ft.} \quad \text{Ans.}$$

(82) (*a*) See Art. **150.**
(*b*) See Art. **150.**
(*c*) See Art. **153.**

(83) See Art. **79.**

(84) See Tables 5 and 6.
$-50-(-37.8)\times 12\times .0333 = 4.875$ = units of heat necessary to raise the temperature to the fusion point. $5.09\times 12 = 61.8$ B. T. U. required to melt the mercury. $[662-(-37.8)]\times .0333\times 12 = 279.64$ B. T. U. required to raise the temperature to the point of vaporization. $157\times 12 = 1,884$ B. T. U. required for vaporizing it. $4.875+61.08+279.64+1,884 = 2,229.595$ B. T. U. Ans.

(85) See Art. **88.**

(*a*) $460+96 = 556°$ F.

(*b*) $273\tfrac{1}{3}+32 = 305\tfrac{1}{3}°$ C.

(*c*) $273\tfrac{1}{3}+180 = 453\tfrac{1}{3}°$ C.

(*d*) $460+650 = 1,110°$ F.

(*e*) $273\tfrac{1}{3}-40 = 233\tfrac{1}{3}°$ C.

(86) (*a*) and (*b*) See Art. **158.**

(87) See Art. **105.**

(88) See Art. **128.**

(89) See Art. **174.**

(90) (*a*) See Art. **129.**
(*b*) See Art. **130.**

(91) (*a*) See Art. **161.**
(*b*) See Art. **162.**

(92) See Art. **142.**

(93) (*a*) and (*b*) See Art. **164.**

(94) See Art. **132.** About 190,000 miles per second.

(95) See Art. **139.**

(96) See Art. **183.**

THEORETICAL CHEMISTRY.

(1) $$\underset{162}{CaH_2(CO_3)_2} = \underset{100}{CaCO_3} + \underset{18}{H_2O} + \underset{44}{CO_2}$$

Using formula **10,** $w = \frac{Wm}{M}$, and substituting the proper values, we obtain

$$w = \frac{296.3 \times 100}{162} = 182.9 \text{ grams.} \quad \text{Ans.}$$

(2) See Arts. **34** and **35.**

(3) See Art. **18.**

(4) See Art. **107.**

(5) (*a*) One atom.
(*b*) Three atoms.
(*c*) Four atoms.

(6) Decant the liquid and evaporate some of it; if the powder was dissolved, it will be found in the evaporating pan after the liquid has been evaporated off.

(7) See Art. **24.**

(8) See Art. **61.**

(9) See Art. **96.**

(10) See Art. **41.** As 35.5 parts of chlorine combine with 1 part of hydrogen, the quantity of antimony that

combines with 35.5 parts of chlorine is its atomic weight, if there is only 1 atom of chlorine in the compound:

$$46.61 : 53.39 = 35.5 : 40.66.$$

By adding the two combining weights $(35.5 + 40.66)$ we obtain only 76.16. This, however, is one-third of the molecular weight (228.5); hence, the compound in question is composed of 3 parts of chlorine and $40.66 \times 3 = 121.98$ parts of antimony, and 121.98, say 122, is the atomic weight of antimony. Ans.

(11) See Arts. **1** and **2.**

(12) See Art. **104.**

(13) See Art. **11.**

(14) Methyl-orange changes its color to reddish pink in the presence of the slightest excess of acid; while phenolphthalein is unaltered by acids, but produces, with the slightest excess of an alkali, an intense magenta-red color.

(15) *K*, *Na*, *Fe*, and *P* are metals; *Cl*, *S*, *N*, *As*, and *H* are non-metals. See Art. **37.**

(16) (*a*) See Art. **52.**
(*b*) See Art. **53.**

(17) Oxidizing agents (see Art. **110**). For reducing agents see Art. **112.**

(18) See Art. **4.**

(19) See Art. **62.**

(20) Chromic acid H_2CrO_4 has a molecular weight of 118.5; using formula **1,** $x = \frac{100\,a\,n}{m}$, and substituting the given values, we obtain

$$x = \frac{1 \times 2 \times 100}{118.5} = 1.69\% \text{ of hydrogen;}$$

$$x = \frac{52.5 \times 1 \times 100}{118.5} = 44.30\% \text{ of chromium;}$$

$$x = \frac{16 \times 4 \times 100}{118.5} = 54.01\% \text{ of oxygen. Ans.}$$

(21) Using formula **11,** $v = \frac{VH}{h}$, and substituting the known values, we obtain

$$v = \frac{542 \times 781}{760} = 556.97 \text{ c.c.} \quad \text{Ans.}$$

(22) See Art. **5.**

(23) By filtration (see Art. **99**), and then by distillation (see Art. **101**) or evaporation (see Art. **98**).

(24) See Art. **8.**

(25) See Art. **37.**

(26) See Art. **6.**

(27) By evaporation (see Art. **98**).

(28) Using formula **2,** $n = \frac{m x}{100 a}$, and substituting the known values, we obtain

$$C = \frac{14.29 \times 84}{12 \times 100} = 1;$$

$$O = \frac{57.14 \times 84}{16 \times 100} = 3;$$

$$H = \frac{1.19 \times 84}{1 \times 100} = 1;$$

$$Na = \frac{27.38 \times 84}{23 \times 100} = 1.$$

Hence, the formula of this compound is CO_3HNa or $HNaCO_3$. Ans.

(29) See Art. **27.**

(30) See Art. **43.**

(31) (*a*) See Art. **75.**

(*b*) See Art. **76.**

(32) $$4CO + 4O_2 = 4CO_2 + 2O_2$$

Four volumes of carbon dioxide will be formed, while two volumes of oxygen remain unburned.

(33) See Arts. **102** and **103.**

(34) See Art. **13.**

(35) See Arts. **55, 56,** and **57.**

(36) See Art. **108.**

(37) See Arts. **14** and **15.**

(38) See Art. **37.**

(39) See Art. **59.**

(40) $Na_2B_4O_7$ has a molecular weight of 202; using formula **1,** $x = \frac{100\,a\,n}{m}$, and substituting the proper values, we obtain

$$x = \frac{23 \times 2 \times 100}{202} = 22.77\% \text{ of sodium;}$$

$$x = \frac{11 \times 4 \times 100}{202} = 21.78\% \text{ of boron;}$$

$$x = \frac{16 \times 7 \times 100}{202} = 55.45\% \text{ of oxygen.} \quad \text{Ans.}$$

(41) (*a*) and (*b*) See Arts. **71** and **72.**

(42) See Art. **16.**

(43) See Art. **58.**

(44) See Art. **78.**

(45) See Art. **26.**

(46) (*a*) See Art. **44.**
(*b*) See Art. **45.**

(47) (*a*) and (*b*) See Art. **66.**

(48) (*a*) See Art. **77.**

(*b*) $A+B = AB$; $AB+C = AC+B$; $AB+CD = AC+BD$; $ABC = CAB$; $AB = A+B$.

(49) $NH_3 + HI = NH_4I$. Using formula **7,** $M = \frac{Wm}{w}$, and then substituting the known values in it, we obtain $M = \frac{56.1 \times 145}{478.5} = 17$, molecular weight of ammonia. Ans.

(50) See Arts. **91, 93,** and **94.**

(51) (*a*) See Art. **19.**

(*b*) See Art. **20.**

(52) (*a*) and (*b*) See Art. **33.**

(53) (*a*) See Art. **50.**

(*b*) 2 pentad and 5 dyad atoms.

(54) See Art. **8.**

(55) Employing formula **2,** $n = \frac{m\,x}{100\,a}$, and substituting the proper values, we obtain

$$H = \frac{0.7 \times 142}{1 \times 100} = 1;$$
$$Na = \frac{32.4 \times 142}{23 \times 100} = 2;$$
$$P = \frac{21.8 \times 142}{31 \times 100} = 1;$$
$$O = \frac{45.1 \times 142}{16 \times 100} = 4.$$

Hence, the formula of this compound, which is hydrodisodium phosphate, is HNa_2PO_4. Ans.

(56) See Art. **21.**

(57) See Arts. **34** and **35.**

(58) Using formula **2,** $n = \frac{m\,x}{100\,a}$, and substituting the known values, we obtain

$$n = \frac{103 \times 53.4}{27.5 \times 100} = Al_2;$$
$$n = \frac{103 \times 46.6}{16 \times 100} = O_3.$$

Hence, the formula of alumina is Al_2O_3. Ans.

(59) $$\underset{170}{2KNO_2} + \underset{98}{H_2SO_4} = \underset{174}{K_2SO_4} + \underset{94}{2HNO_2}$$

Using formula **8,** $W = \frac{Mw}{m}$, and substituting the given values, we obtain

$\frac{170 \times 392}{174}$ = 383 grams of potassium nitrite. Ans.

(60) See Art. **30.**

(61) SrO, strontium oxide; $NaCl$, sodium chloride; P_2O_5, phosphoric oxide; B_2O_3, boric oxide; $SnCl_4$, stannic chloride.

(62) Using formula **3,** $a = \frac{m\,x}{100\,n}$, and substituting the given values, we obtain

$$a = \frac{101\times 38.7}{1\times 100} = 39, \text{ molecular weight of potassium. Ans.}$$

(63) See Art. **65.**

(64) See Art. **38.**

(65) See Art. **30.**

(66) See Art. **61.**

(67) Substituting the known values in formula **3,** $a = \frac{m\,x}{100\,n}$, we obtain

$$a = \frac{125\times 52}{1\times 100} = 65, \text{ molecular weight of zinc. Ans.}$$

(68) Using formula **12,** $v_1 = v\left(\frac{273+t_1}{273+t}\right)$, and substituting the given values, we obtain

$$v_1 = 40.1\left(\frac{273+0}{273+100}\right) = 29.349 \text{ c. c. Ans.}$$

(69) See Art. **33.**

(70) (*a*) and (*b*) See Arts. **46, 47, 48,** and **49.**

(71) See Art. **70.**

(72) Using formula **4,** $m = \frac{100\,a\,n}{x}$, and substituting the given values, we obtain

$$m = \frac{65\times 1\times 100}{67} = 97, \text{ molecular weight of zinc sulphide.}$$

Ans.

(73) Using formula **7,** $M = \frac{Wm}{w}$, and substituting the known values, we obtain

$M = \frac{883.2 \times 162}{1{,}117.8} = 128$, molecular weight of hydriodic acid. Ans.

(74) By employing formula **12,** $v_1 = v\left(\frac{273 + t_1}{273 + t}\right)$, and substituting the given values, we obtain

$$v_1 = 350\left(\frac{273 + 94}{273 + 74}\right) = 370.17 \text{ c. c.} \quad \text{Ans.}$$

(75) See Art. **29.**

(76) 1 atom of hydrogen, 1 atom of oxygen, 1 molecule of oxygen, 3 molecules of hydrogen.

(77) 11.2 liters of hydrogen weigh 1 gram.

(78) Substituting the given values in formula **1,** $x = \frac{a\,n \times 100}{m}$, we obtain

$$\frac{44 \times 1 \times 100}{88} = 55 \text{ parts of } N_2O. \quad \text{Ans.}$$

(79) Using formula **9,** $m = \frac{Mw}{W}$, and substituting the given values, we obtain

$m = \frac{228.5 \times 2 \times 78.57}{105.6} = 340$, molecular weight of antimonous sulphide. Ans.

(80) Add to the dilute acid a drop of litmus solution, and then caustic soda in a solution of known strength in small quantities, until the red tint, caused by the previous addition of the litmus, has changed to a faint-blue color. See also Arts. **113** to **118.**

(81) See Art. **30.**

(82) See Art. **40.**

(83) Using formula **4,** $m = \frac{100\,a\,n}{x}$, and substituting the known values, we obtain

$m = \frac{23 \times 1 \times 100}{39.32} = 58.5$, molecular weight of $NaCl$. Ans.

(84) $$\underset{98}{H_2SO_4} + \underset{106}{Na_2CO_3} = \underset{142}{Na_2SO_4} + \underset{44}{CO_2} + \underset{18}{H_2O}$$

Using formula **10,** $w = \frac{Wm}{M}$, and substituting the given values, we obtain

$w = \frac{396.11 \times 142}{106} = 530.64$ grams of sodium sulphate. Ans.

(85) (*a*) See Arts. **26** and **63.** (*b*) It means that the volume has to be calculated at normal temperature and pressure; i. e., at 0° and 760 mm. pressure.

(86) (*a*) See Art. **69.**

(*b*) See Art. **69.**

(87) Using formula **6,** $z = \frac{m \times y}{a\,n}$, and substituting the known values, we obtain

$z = \frac{310 \times 501}{31} = 5{,}010$ kilograms of calcium phosphate. Ans.

(88) Using formula **11,** $v = \frac{VH}{h}$, and substituting the known values, we obtain

$$v = \frac{420 \times 739}{760} = 408.4 \text{ c. c.} \quad \text{Ans.}$$

(89) (*a*) See Art. **55.** (*b*) Sulphurous acid has the formula H_2SO_3, while sulphuric acid is H_2SO_4; hence, sulphuric acid possesses one more atom of oxygen than does sulphurous acid.

(90) (*a*) See Art. **73.**

(*b*) $AB = A + B$; $A + B = AB$; $AB + CD = AD + BC$.

(91) One liter of hydrogen weighs .0896 gram.

(92) See Art. **68.**

(93) Potassium iodide KI, lead sulphide PbS, calcium chloride $CaCl_2$, cupric oxide CuO.

(94)
$$\underset{202}{2KNO_3} + \underset{98}{H_2SO_4} = \underset{174}{K_2SO_4} + \underset{126}{2HNO_3}$$

202 parts of potassium nitrate and 98 parts of sulphuric acid yield 174 parts of potassium sulphate and 126 parts of nitric acid.

(95) Using formula **5**, $y = \frac{an \times z}{m}$, and substituting the known values, we obtain

$$y = \frac{207 \times 932}{239} = 807.21 \text{ Kg. of lead. Ans.}$$

INORGANIC CHEMISTRY.

(PART 1.)

---

(1) See Art. **125.**

(2) See Art. **4.**

(3) 1 kilogram = 1,000 grams; 50 kilograms = 50,000 grams. See Art. **5.** 1 gram of burning hydrogen raises 34,462 grams of water from 0° to 1°. Then, 50,000 ÷ 34,462 = 1.45 grams of hydrogen are needed to raise 50,000 grams from 0° to 1°, and 1.45 × 10, or 14.5, grams will be required to raise the same quantity of water to 10°. Ans.

(4) See Art. **11.** Oxygen is an extremely active supporter of combustion, while air, being composed of oxygen and nitrogen, the latter of which is not a supporter of combustion, is a less active supporter.

(5) See Art. **100.**

(6) See Art. **61.**

(7) $2NaBr + Cl_2 = 2NaCl + Br_2$.

(8) See Art. **36.**

(9) All acids, all hydrates, organic compounds, such as alcohol, petroleum, etc. See Art. **2.**

(10) Hydrochloric-acid gas is prepared according to the equation

$$\underset{98}{H_2SO_4} + \underset{117}{2NaCl} = \underset{73}{2HCl} + \underset{142}{Na_2SO_4}$$

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Then,

(*a*) $73 : 98 = 36.5 : 49$ grams H_2SO_4;
$73 : 117 = 36.5 : 58.5$ grams $NaCl$.

(*b*) $73 : 98 = 1 : 1.3424$ kilograms H_2SO_4;
$73 : 117 = 1 : 1.6027$ kilograms $NaCl$. Ans.

(11) See Art. **71.**

(12) See Art. **44.**

(13) See Art. **115.**

(14) See Art. **99.**

(15) See Art. **85.**

(16) (*a*) See Art. **3** and Art. **31,** Experiment 22.

(*b*) A molecule of water contains 2 parts of hydrogen and 16 parts of oxygen, or 1 gram of water contains $\frac{2}{18}$ or $\frac{1}{9}$ part of its weight of hydrogen:

$$250 \div 9 = 27.777+ \text{ grams of } H. \quad \text{Ans.}$$

(17) See Arts. **30** and **31.**

(18) See Art. **98.**

(19) See Fig. 13 and Art. **22.**

(20) See Arts. **4** and **5.**

(21) See Art. **40.**

(22) (*a*) No.

(*b*) See Art. **75.**

(23) See Art. **28.**

(24) See Art. **10.** Oxygen is prepared according to the equation

$$\underset{261}{3MnO_2} = \underset{229}{Mn_3O_4} + \underset{32}{O_2}$$

Then,

$$261 : 32 = 261 : 32 \text{ grams.} \quad \text{Ans.}$$

(25) See Art. **55.**

(26) $H_2SO_4 + Zn = ZnSO_4 + H_2$. See Art. **3.**

(27) See Art. **10.**

(28) (*a*) See Art. **39.**
(*b*) See Art. **41.**

(29) (*a*) See Art. **45.**
(*b*) See Figs. 23 and 24.

$$4HCl + MnO_2 = MnCl_2 + 2H_2O + Cl_2$$
$$2NaCl + 2H_2SO_4 + MnO_2 = MnSO_4 + Na_2SO_4 + 2H_2O + Cl_2$$

(30) A desiccating agent is a compound that possesses a strong affinity for water or moisture; as, for instance, sulphuric acid, calcium chloride, etc. See also Art. **5.**

(31) (*a*) See Art. **116.**
(*b*) See Art. **117.**

(32) H_2SeO_4. See Art. **139.**

(33) See Art. **5.**

(34) Nascent hydrogen is distinguished by its increased chemical activity. See Art. **108,** *Theoretical Chemistry.*

(35) Silver has an atomic weight of 108; chlorine has an atomic weight of 35.5. Molecular weight of $AgCl$ = 143.5. Using formula **1,** *Theoretical Chemistry,*

$$x = \frac{100an}{m},$$

and substituting the given values, we obtain

$$x = \frac{100 \times 108 \times 1}{143.5} = 75.261\% \text{ of silver,}$$

$$x = \frac{100 \times 35.5 \times 1}{143.5} = 24.739\% \text{ of chlorine. Ans.}$$

(36) See Art. **10.** The production of oxygen from mercuric oxide is expressed by the equation

$$\underset{431.6}{2HgO} = \underset{399.6}{2Hg} + \underset{32}{O_2}$$

Then,

431.6 : 32 = 20 : 1.48 grams of oxygen. Ans.

(37) $2Na + 2H_2O = 2NaHO + H_2$. See Art. **3.**

(38) $5H_2 + 3O_2 = 5H_2O + O$.

(39) Two molecules of water and 1 molecule of potassium yield 2 molecules of potassium hydrate and 1 molecule of hydrogen.

(40) See Art. **61.**

(41) See Art. **76.**

(42) 114.7 grams.

(43) See Art. **5.**

(44) See Art. **97.**

(45) $H_2S_2O_7$

(46) See Art. **56.**

(47) Hydrogen is obtained according to the equation

$$\begin{array}{ccccccc} H_2SO_4 & + & Zn & = & ZnSO_4 & + & H_2 \\ 98 & & 65 & & 161 & & 2 \end{array}$$

10 liters of hydrogen weigh .896 gram; then,

2 : 65 = .896 : 29.120 grams of zinc,

2 : 98 = .896 : 43.904 grams of sulphuric acid. Ans.

(48) Chlorine, bromine, iodine, and fluorine. See Art. **73.**

(49) (*a*) Water, sulphur dioxide, phosphorus pentoxide.

(*b*)
$$\begin{aligned} 2H_2 + O_2 &= 2H_2O \\ S + O_2 &= SO_2 \\ P_4 + 5O_2 &= 2P_2O_5 \end{aligned}$$

(50) Bromine colors a solution of starch yellow, while iodine gives a blue color to the solution. See also Arts. **77** and **80.**

(51) Potassium chlorate has the formula $KClO_3$; its molecular weight is 122.5. Using formula **1,** *Theoretical Chemistry*,

$$x = \frac{100\,a\,n}{m},$$

and substituting the known values, we obtain

$$x = \frac{100 \times 39 \times 1}{122.5} = 31.84\%\ K,$$

$$x = \frac{100 \times 35.5 \times 1}{122.5} = 28.98\%\ Cl,$$

$$x = \frac{100 \times 16 \times 3}{122.5} = 39.18\%\ O. \quad \text{Ans.}$$

(52) 3.17 grams occupy the volume of 1 liter. 1 gram occupies, then, the volume of .31545 liter, and 32 grams will occupy 10.094 liters. Ans.

(53) One molecule of zinc and 1 molecule of sulphuric acid yield 1 molecule of zinc sulphate and 1 molecule of hydrogen.

(54)

Hyposulphurous oxide	$S''O$	Hyposulphurous acid	$H_2S''O_2$
Sulphurous oxide	$S^{iv}O_2$	Sulphurous acid	$H_2S^{iv}O_3$
Sulphuric oxide	$S^{vi}O_3$	Sulphuric acid	$H_2S^{vi}O_4$

(55) See Art. **10.**

(56) See Art. **88.**

(57) Hydrogen sulphide has the formula H_2S; its molecular weight is 34. Using formula **10,** *Theoretical Chemistry,*

$$w = \frac{Wm}{M},$$

and substituting the known values, we obtain

$$w = \frac{100 \times 32}{34} = 94.117 \text{ grams.} \quad \text{Ans.}$$

(58) 1 liter of H_2S weighs 1.52 grams, 100 liters weigh 152 grams.

(59) (*a*) and (*b*) See Art. **90.**

(60) 16 parts, by weight, of oxygen,
2 parts, by weight, of hydrogen.

For the determination of the composition of water by weight, see Art. **34.**

(61) See Art. **99.**

(62) See Art. **48.**

(63) See Art. **10,** Experiment 7, and Figs. 6 and 7.

(64) See Art. **108.**

(65) Hydrofluoric acid has the formula HF and a molecular weight of 20. Using formula **1,** *Theoretical Chemistry,*

$$x = \frac{100\,a\,n}{m},$$

and substituting the known values, we obtain:

$$x = \frac{100 \times 1 \times 1}{20} = 5\% \text{ of } H,$$

$$x = \frac{100 \times 19 \times 1}{20} = 95\% \text{ of } F. \quad \text{Ans.}$$

(66) See Art. **145.**

(67) See Art. **113.**

(68) (*a*) $2NaI + MnO_2 + 2H_2SO_4$
$= Na_2SO_4 + MnSO_4 + 2H_2O + I_2$.

Two molecules of sodium iodide and 1 molecule of manganese dioxide and 2 molecules of sulphuric acid yield 1 molecule of sodium sulphate, 1 molecule of manganese sulphate, 2 molecules of water, and 1 molecule of iodine. See Art. **79.**

(69) See Art. **134.**

(70) (*a*) A unit of heat is the amount of heat required to raise 1 gram of water from 0° to 1°.

(*b*) 34,462 units of heat. See Art. **5.**

(71) See Art. **99.**

(72) A reducing agent is a substance that removes oxygen, or elements similar to it, from its compounds, or decreases the valence of a substance, changing it from a higher to a lower state of oxidation.

(73) (*a*) See Art. **122.**

(*b*) SO_2.

(74) (*a*) and (*b*) See Art. **16.**

(75) The antiseptic or antizymotic property of a substance is that property which prevents or destroys putrefaction.

(76) (*a*) See Art. **54,** and Fig. 28.

(*b*) $NaCl + H_2SO_4 = HCl + NaHSO_4$

or with the application of heat,

$$2NaCl + H_2SO_4 = 2HCl + Na_2SO_4$$

(77) (*a*) See Art. **123.**

(*b*) See Art. **129.**

(78) See Art. **47.**

(79) See Art. **11.**

(80) A suitable freezing mixture is prepared by mixing 2 parts of ice with 1 part of ordinary salt.

(81) See Art. **105.**

(82) See Art. **14,** Experiment 17.

(83) (*a*) See Art. **77.**

(*b*) See Arts. **76** and **77.**

(84) See Art. **126.**

(85) See Art. **128.**

(86) See Art. **109.**

(87) See ozone, Arts. **13** and **16.**

(88) See Art. **127.**

(89) SO_2 is obtained according to the equation

$$\underset{64}{S_2} + \underset{64}{2O_2} = \underset{128}{2SO_2}$$

Using formula **10,** *Theoretical Chemistry,*

$$w = \frac{Wm}{M},$$

and substituting the known values, we obtain

$$w = \frac{10 \times 128}{64} = 20 \text{ grams of } SO_2. \quad \text{Ans.}$$

INORGANIC CHEMISTRY.

(PART 2.)

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(1) $$\underset{\substack{\text{calcium}\\\text{fluoride}}}{3CaF_2}+\underset{\substack{\text{boric}\\\text{oxide}}}{B_2O_3}+\underset{\substack{\text{sulphuric}\\\text{acid}}}{3H_2SO_4} = \underset{\substack{\text{calcium}\\\text{sulphate}}}{3CaSO_4}+\underset{\text{water}}{3H_2O}+\underset{\substack{\text{boron}\\\text{fluoride}}}{2BF_3}$$

(2) (*a*) For the preparation of carbon dioxide see Art. **119.**

(*b*) $$\underset{100}{CaCO_3} = \underset{56}{CaO}+\underset{44}{CO_2}$$

Using formula **6,** Art. **84,** *Theoretical Chemistry,*

$$z = \frac{m \times y}{a\,n},$$

we obtain $z = \frac{100 \times 594}{44} = 1{,}350$ grams of calcium carbonate. Ans.

Or, $$\underset{100}{CaCO_3}+\underset{73}{2HCl} = \underset{111}{CaCl_2}+\underset{18}{H_2O}+\underset{44}{CO_2}$$

$z = \frac{100 \times 594}{44} = 1{,}350$ grams of calcium carbonate. Ans.

(3) See Table 1, Art. **7.**

(4) (*a*) Using formula **2,** Art. **82,** *Theoretical Chemistry,*

$$n = \frac{m\,x}{100\,a},$$

and substituting the known values, we obtain

$$n = \frac{108 \times 25.93}{100 \times 14} = 2N, \qquad n = \frac{108 \times 74.07}{100 \times 16} = 5O.$$

Its formula, therefore, is N_2O_5. Ans.

(*b*) Nitric oxide, etc. See Art. **32.**

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(5) Calcium carbonate has the formula

$$\begin{array}{c} CaCO_3 \\ \underbrace{40+12+48}_{\text{100 molecular weight,}} \end{array}$$

which shows that calcium carbonate contains 40% of calcium; hence, 1,000 kilograms of $CaCO_3$ must contain 400 kilograms of calcium. Ans.

(6) See Art. **148.**

(7) Ammonia has the formula NH_3, and a molecular weight of 17. Using formula **1,** Art. **81,** *Theoretical Chemistry,*

$$x = \frac{100\,a\,n}{m},$$

we obtain
$$x = \frac{100 \times 14 \times 1}{17} = 82.353\%\ N,$$
$$x = \frac{100 \times 1 \times 3}{17} = 17.647\%\ H. \quad \text{Ans.}$$

(8) See Art. **3.**

(9) Cyanogen; the specific gravity and characteristic odor are mentioned in Art. **155,** and the density is given (in Italics) below the heading "Cyanogen."

(10) See Art. **134.**

(11) $CuHAsO_3$.

(12) See Art. **46.**

(13) One liter of nitrogen, i. e., 1,000 c. c., weighs 1.255 grams; 250 c. c. must, therefore, weigh .31375 gram. Ans.

(14) (*a*) See Art. **105.**
(*b*) See Art. **106.**

(15) $SiCl_4 + 4H_2O = H_4SiO_4 + 4HCl.$

(16) See Art. **8.**

(17) $(OH)H_2N.$

(18) H_2NaPO_4, dihydrosodium phosphate.
$HCaPO_4$, hydrocalcium phosphate.
$BiPO_4$, bismuth phosphate.

(19) (*a*) Arsenate of soda, $Na_2HAsO_4,7Aq$.
(*b*) Calcium arsenate, $CaHAsO_4,7Aq$.

(20) See Art. **114.**

(21) See Art. **141.**

(22) Carbon disulphide is formed according to the equation:

$$\underset{24}{C_2} + \underset{128}{2S_2} = \underset{152}{2CS_2}$$

Using formula **8,** Art. **85,** *Theoretical Chemistry,*

$$W = \frac{Mw}{m},$$

and substituting the proper values, we obtain

$$W = \frac{64 \times 76}{76} = 64 \text{ grams of } S. \quad \text{Ans.}$$

(23) Borax has the formula $Na_2B_4O_7$; its molecular weight, consequently, is 202. Using formula **1,** Art. **81,** *Theoretical Chemistry,*

$$x = \frac{100\,a\,n}{m},$$

we obtain

$$x = \frac{100 \times 23 \times 2}{202} = 22.77\%\ Na,$$

$$x = \frac{100 \times 11 \times 4}{202} = 21.78\%\ B,$$

$$x = \frac{100 \times 16 \times 7}{202} = 55.45\%\ O. \quad \text{Ans.}$$

(24) H_3PO_4 is phosphoric acid. See Art. **59** for its properties. PH_3 is phosphine. See Art. **54** for its properties.

(25) Nitrogen dioxide is obtained according to the equation:

$$\underset{189}{3Cu} + \underset{504}{8HNO_3} = \underset{561}{3Cu(NO_3)_2} + \underset{60}{N_2O_2} + \underset{72}{4H_2O}$$

and, as 1 liter of nitrogen dioxide weighs 1.33 grams, 100 liters weigh 133 grams.

As 60 : 189 :: 133 : 418.95
so is 60 : 504 :: 133 : 1,117.2

from which it follows that, in order to produce 100 liters of nitrogen dioxide, 418.95 grams of copper and 1,117.2 grams of nitric acid are required. Ans.

(26) See Art. **38.**

(27) (*a*) and (*b*) See Art. **74.**

(28) See Art. **90.**

(29) See Art. **121.**

(30) (*a*) See Art. **102.**
(*b*) See Art. **103.**

(31) See Art. **138.**

(32) See Art. **154,** Experiment 41.

(33) Carbon dioxide is formed according to the equation:

$$\underset{12}{C} + \underset{32}{O_2} = \underset{44}{CO_2}$$

Using formula **5,** Art. **84,** *Theoretical Chemistry*,

$$y = \frac{a\,n \times z}{m},$$

and substituting the proper values, we obtain

$$y = \frac{44 \times 48}{12} = 176 \text{ grams of } CO_2. \quad \text{Ans.}$$

(34) See Art. **14.**

(35) See Art. **15.**

(36) Nitrogen, with which oxygen is mixed in air, is a very inert gas, and does not support combustion.

(37) See Art. **78.**

(38) From the question, there are 23 grams of O in 100 grams of air. The chemical action when P burns in O is represented in the following equation:

$$\underset{62}{2P} + \underset{80}{5O} = \underset{142}{P_2O_5}$$

To burn 80 by weight of O, 62 of P are required; then, the number of grams of P required to burn 23 grams of O is determined by the proportion

$$80 : 23 :: 62 : 17.8. \quad \text{Ans.}$$

(39) $CO_2 + H_2O = H_2CO_3$.

(40) $CuO + CO = Cu + CO_2$.

(41) $\underset{12}{C} + \underset{32}{O_2} = \underset{44}{CO_2}$.

As 12 parts of C require 32 parts of O, 1,000 kilograms will require 2,666.6 kilograms of O; and, as 1 liter of oxygen, i. e., 1 cubic decimeter, weighs 1.429 grams, 1,000 cubic decimeters, or 1 cubic meter, weigh 1,429 grams, or

1.429 kilograms of O = 1 cubic meter;
hence, 2,666.6 kilograms of O = 1,866.06 cubic meters. Ans.

(42) Ammonia is formed according to the equation:

$$\underset{107}{2NH_4Cl} + \underset{56}{CaO} = \underset{111}{CaCl_2} + \underset{18}{H_2O} + \underset{34}{2NH_3}$$

Employing formula **8**, Art. **85**, *Theoretical Chemistry*,

$$W = \frac{Mw}{m},$$

and substituting the proper values (necessarily taking 34 for m), we obtain

$$W = \frac{56 \times 59}{34} = 97.17 \text{ grams of } CaO. \quad \text{Ans.}$$

(43) The formula of ethylene gas is H_4C_2; its molecular weight is 28. Using formula **1**, Art. **81**, *Theoretical Chemistry*,

$$x = \frac{100\,a\,n}{m},$$

we obtain

$$H = \frac{100 \times 1 \times 4}{28} = 14.28\% \text{ of } H,$$

$$C = \frac{100 \times 12 \times 2}{28} = 85.72\% \text{ of } C.$$

(44) See Art. **122**, *Theoretical Chemistry*, and Art. **142**, of this paper.

(45) See Art. **157**.

(46) $Si + 3HCl = SiHCl_3 + H_2$.

(47) Referring to Art. **14**, it will be seen that (*a*) at 0° one volume of water absorbs 1,149 volumes of ammonia gas;

consequently, 3.92 liters will absorb 4,504.08 liters; (*b*) at 15° one volume of water absorbs 783 volumes of ammonia gas, or 3.92 liters will absorb 3,069.36 liters.

(48) N_2O, or hyponitrous oxide, is prepared according to the following equation (see Art. **24**):

$$\underset{630}{10HNO_3} + \underset{260}{4Zn} = \underset{44}{N_2O} + \underset{756}{4Zn(NO_3)_2} + \underset{90}{5H_2O}$$

Using formula **10,** Art. **85,** *Theoretical Chemistry,*

$$w = \frac{Wm}{M},$$

and substituting the proper values, we obtain

$$w = \frac{.44 \times 1}{260} = .169 \text{ kilogram of } N_2O;$$

and, as 1.97 grams of hyponitrous oxide occupy the volume of 1 liter, .169 kilo, or 169 grams, occupy the volume of 85.787 liters. Ans.

(49) See Art. **40.**

(50) See Arts. **47** and **48.**

(51) The best antidotes, in case of poisoning from arsenic, are freshly prepared ferric or magnesic hydrate, which form insoluble compounds with the arsenic and thus render this poison comparatively harmless.

(52) See Art. **94.**

(53) Carbon monoxide is obtained from oxalic acid according to the equation:

$$\underset{90}{H_2C_2O_4} = \underset{18}{H_2O} + \underset{44}{CO_2} + \underset{28}{CO}$$

Using formula **10,** Art. **85,** *Theoretical Chemistry,*

$$w = \frac{Wm}{M},$$

and substituting the known values, we obtain

$$\frac{100 \times 28}{90} = 31.11 \text{ grams of } CO.$$

1.25 grams of CO occupy the volume of 1 liter; and, consequently, 31.11 grams occupy the volume of 24.88 liters. Ans.

(54) See Art. **156.**

(55) Nitric acid is obtained by the action of sulphuric acid on sodium nitrate, according to the equation:

$$\underset{85}{NaNO_3} + \underset{98}{H_2SO_4} = \underset{63}{HNO_3} + \underset{120}{NaHSO_4}$$

Using formula **10,** Art. **85,** *Theoretical Chemistry,*

$$w = \frac{Wm}{M},$$

and substituting the known values, we obtain

$$w = \frac{63 \times 170}{85} = 126 \text{ grams of nitric acid. Ans.}$$

(56) As 100 c. c. of ammonia give 200 c. c. of a mixture of hydrogen and nitrogen, i. e., 150 c. c. of H and 50 c. c. of N (see Art. **17**), 250 c. c. of ammonia, on being decomposed by electric sparks, will give 500 c. c. of mixed gases.

(57) Phosphorus pentoxide is obtained synthetically by burning phosphorus in oxygen, according to the equation:

$$\underset{124}{P_4} + \underset{160}{5O_2} = \underset{284}{2P_2O_5}$$

Using formula **5** or formula **10,** Arts. **84** and **85,** *Theoretical Chemistry,*

$$y = \frac{an \times z}{m},$$

and substituting the known values, we obtain

$$y = \frac{284 \times 793}{124} = 1{,}816.22 \text{ grams of phosphorus pentoxide.}$$

Ans.

(58) (*a*) See Art. **39.**

(*b*) Two hundred and fifty c. c. of nitric acid and 750 c. c. of hydrochloric acid.

(59) See Arts. **58** and **59.**

(60) See Art. **100.**

(61) Carbonic acid is a dibasic acid, because it contains two replaceable atoms of H.

(62) See Art. **144.**

(63) See Art. **165.**

(64) The equations are

$$\underset{73}{2HCl}+\underset{65}{Zn} = \underset{136}{ZnCl_2}+\underset{2}{H_2}$$

$$\underset{73}{2HCl}+\underset{100}{CaCO_3} = \underset{111}{CaCl_2}+\underset{18}{H_2O}+\underset{44}{CO_2}$$

An inspection of the equation shows that 2 molecules of the hydrochloric acid cause, respectively, the evolution of 1 molecule of hydrogen H_2 and 1 molecule of carbon dioxide; and the volume of gases must, according to Avogadro's law, be the same, and must be one-half the volume of the hydrochloric acid used.

As 1 liter of hydrochloric acid weighs 1.63 grams, 10 grams occupy 6.135 liters, and the hydrogen and carbon dioxide evolved each occupy 3.0675 liters. Ans.

(65) See Art. **41.**

(66) See Art. **71.**

(67) See Art. **109.**

(68) See Art. **120.**

(69) Cyanogen is prepared from cyanide of mercury, according to the equation:

$$\underset{252}{Hg(CN)_2} = \underset{200}{Hg}+\underset{52}{(CN)_2}$$

Using formula **6,** Art. **84,** *Theoretical Chemistry,*

$$z = \frac{m \times y}{a\, n},$$

and substituting the known values, we obtain

$$z = \frac{252 \times 50}{52} = 242.308 \text{ grams. Ans.}$$

(70) N_2O, 28 parts of N and 16 parts of O.
N_2O_2, 28 parts of N and 32 parts of O.
N_2O_3, 28 parts of N and 48 parts of O.
N_2O_4, 28 parts of N and 64 parts of O.
N_2O_5, 28 parts of N and 80 parts of O.

(71) (*a*) The formula of phosphine is H_3P.

(*b*) See Art. **53.**

(*c*) The reaction—a somewhat complicated one—may be expressed thus:

$$P_4+3KOH+3H_2O = 3KH_2PO_2+H_3P$$

(72) No; the reason for this is given in that part of Art **43** which refers to Experiment 5.

(73) Whether this white powder is arsenious oxide, or at least contains some, may be ascertained by sprinkling it upon a red-hot coal. If a garlic-like odor is perceptible, this may be taken as an indication that the powder in question contains arsenious oxide.

(74) From the given equation:

$$CH_4+2O_2 = CO_2+2H_2O$$

we find that 1 volume of marsh gas requires 2 volumes of oxygen; therefore, 100 measures will require 200 measures of pure oxygen. Air contains 21%, by volume, of oxygen; therefore,

21 : 200 :: 100 : 952 volumes of pure air.

(75) (*a*) See Art. **111.**

(*b*) See Art. **112.**

(76) Carbon disulphide is obtained according to the following equation:

$$\underset{24}{C_2}+\underset{128}{2S_2} = \underset{152}{2CS_2}$$

1 liter of carbon-disulphide vapor weighs 3.40 grams; the desired 1.25 liters, consequently, weigh 4.25 grams. Using formula **8,** Art. **85,** *Theoretical Chemistry,*

$$W = \frac{Mw}{m},^{*}$$

and substituting the proper values, we obtain

$$W = \frac{128\times4.25}{152} = 3.578 \text{ grams of } S. \quad \text{Ans.}$$

---

[*] In stating these formulas in *Theoretical Chemistry,* it has, of course, been assumed that each molecule of the factor yielded one of the products. If in any reaction this is not true, then M and m must represent the sum of the molecular weights expressed in the equation.

(77) Hyponitrous oxide, also known as "laughing gas," is widely used in surgery and dentistry (see Art. **25**).

(78) See Art. **70.**

(79) Methane is obtained from sodium acetate (see Art. **103**) according to the equation:

$$\underset{82}{NaC_2H_3O_2} + \underset{40}{NaOH} = \underset{106}{Na_2CO_3} + \underset{16}{H_4C}$$

Employing formula **10,** Art. **85,** *Theoretical Chemistry,*

$$w = \frac{Wm}{M},$$

and substituting the proper values, we obtain

$$w = \frac{888 \times 16}{82} = 173.26 \text{ grams of methane;}$$

and, as 1 liter of methane weighs .715 gram, 173.26 grams would occupy the volume of 242.32 liters. Ans.

(80) Carbon monoxide does not render lime water milky.

(81) KH_2PO_2.

(82) (*a*) and (*b*) See Art. **162.**

(83) N_2O is obtained from ammonium nitrate according to the equation:

$$\underset{80}{(NH_4)NO_3} = \underset{44}{N_2O} + \underset{36}{2H_2O}$$

Employing formula **5,** Art. **84,** *Theoretical Chemistry,*

$$y = \frac{an \times z}{m},$$

and substituting the proper values, we obtain

$$y = \frac{44 \times 15}{80} = 8.25 \text{ grams;}$$

and, as 1.97 grams of N_2O occupy the volume of 1 liter, 8.25 grams will occupy the volume of 4.188 liters. Ans.

(84) See Art. **152.**

(85) (*a*) and (*b*) See Art. **127.**

(*c*) See Arts. **127, 128, 129,** and **130.**

(86) See Art. **120.**

(87) See Art. **79.**

(88) Either by subjecting it to a pressure of 6½ atmospheres at 10° or by cooling it to −40°.

(89) See Art. **45.**

(90) Arsenic disulphide, or realgar, As_2S_2.
Arsenic trisulphide, or orpiment, As_2S_3.
Arsenic pentasulphide, As_2S_5.

(91) (*a*) See Art. **160.**
(*b*) SiO_2.

(92) Arsenious oxide, As_2O_3.
Arsenic oxide, As_2O_5.

(93) See Art. **58** and Experiment **13.**

(94) See Art. **7.**

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(1) Sc_2O_3.

(2) (1) $$\underset{142}{Na_2SO_4} + \underset{24}{C_2} = \underset{78}{Na_2S} + \underset{88}{2CO_2}$$

(2) $$\underset{78}{Na_2S} + \underset{100}{CaCO_3} = \underset{106}{Na_2CO_3} + \underset{72}{CaS}$$

Using formula **10,** Art. **85,** *Theoretical Chemistry,* we obtain

for (*a*) $$w = \frac{71 \times 78}{142} = 39;$$

$$w = \frac{39 \times 106}{78} = 53 \text{ Kg.} \quad \text{Ans.}$$

For (*b*) $$w = \frac{426 \times 78}{142} = 234;$$

$$w = \frac{234 \times 106}{78} = 318 \text{ Kg.} \quad \text{Ans.}$$

(3) See Art. **28.**

(4) See Art. **2.**

(5) See Art. **151.**

(6) $CaC_2 + 2H_2O = Ca(OH)_2 + C_2H_2$. One molecule of calcium carbide plus 2 molecules of water = 1 molecule of calcium hydrate plus 1 molecule of acetylene.

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(7) $Ca(OH)_2$.

(8) A saturated solution of sodium chloride boils at 109.7°. See Art. **70.**

(9) See Art. **13.**

(10)

$$\underset{\underbrace{69\quad 27.5\quad 114}_{210.5}}{Na_3\ \ Al\ \ F_6}$$

Using formula **1,** Art. **81,** *Theoretical Chemistry,*

$$x = \frac{100\,a\,n}{m},$$

and substituting the proper values, we obtain

$$x = \frac{100 \times 27.5 \times 1}{210.5} = 13.064\%\ Al. \quad \text{Ans.}$$

(11) See Art. **20.**

(12) The spectrum of lithium is characterized by an intense crimson line, having a wave length of .0006705 mm.

(13) See Art. **34.**

(14) Acetylene is obtained according to the formula:

$$\underset{\underbrace{40+24\quad (2+16)2}_{64\ +\ 36}}{CaC_2 + 2H_2O} = \underset{\underbrace{40+34\qquad 24+2}_{74\ +\ 26}}{Ca(OH)_2 + C_2H_2}$$

We further know that 1 liter of C_2H_2 weighs 1.16 grams; hence, 29 liters weigh 33.64 grams.

Employing, now, formula **8,** Art. **85,** *Theoretical Chemistry,*

$$W = \frac{Mw}{m},$$

we obtain $W = \frac{64 \times 33.64}{26} = 82.806$ g. Ans.

(15) Artificial barium sulphate. See Art. **112.**

(16) See Arts. **14, 15,** and **16.**

(17) See Art. **46.**

(18) Metallic carbonates, with the exception of the alkaline carbonates, are insoluble in pure water. See Art. **38.**

(19) See Art. **2.**

(20) Sodium carbonate has the formula Na_2CO_3 and a molecular weight of 106.

Using formula **1,** Art. **81,** *Theoretical Chemistry,*

$$x = \frac{100an}{m},$$

$$\text{we obtain}\quad \text{sodium} = \frac{100\times 23\times 2}{106} = 43.396\%;$$

$$\text{carbon} = \frac{100\times 12\times 1}{106} = 11.321\%;$$

$$\text{oxygen} = \frac{100\times 16\times 3}{106} = 45.283\%.\quad \text{Ans.}$$

(21) NH_4HCO_3.

(22) Valence II.

(23) See Art. **42.**

(24) $3Cl_2 + 6KOH = KClO_3 + 5KCl + 3H_2O.$

(25) See Art. **170.**

(26) See Art. **2.**

(27) Symbol *In.* Atomic weight 113.4. Valence II and III.

(28) Barium occurs in nature as barium carbonate in the mineral *witherite*, and as barium sulphate in *heavy spar*, or *barites.* See Art. **104.**

(29) See Art. **26.**

(30) Magnesia, or magnesium oxide, has the formula MgO.

(31)

$$\underset{100}{CaCO_3} = \underset{44}{CO_2} + \underset{56}{CaO}$$

Using formula **10,** Art. **85,** *Theoretical Chemistry,*

$$w = \frac{Wm}{M}$$

$$w = \frac{139 \times 56}{100} = 77.84 \text{ Kg. Ans.}$$

(32) See Art. **84.**

(33) See Art. **13.**

(34) G_2O_3.

(35) Symbol *Li*. Atomic weight 7.01. For its properties see Art. **98.**

(36) (*a*) and (*b*) See Art. **39.**

(37) $K_2CO_3 + H_2C_4H_4O_6 = KHC_4H_4O_6 + KHCO_3$.

(38) (*a*) and (*b*) See Art. **17.**

(39) (*a*) Zinc blende has the formula ZnS, and a molecular weight of 97. Employing formula **1,** Art. **81,** *Theoretical Chemistry,*

$$x = \frac{100\,a\,n}{m},$$

we obtain $x = \frac{100 \times 65 \times 1}{97} = 67\%$. Ans.

(*b*) Smithsonite has the formula $ZnCO_3$ and a molecular weight of 125. Employing the same formula, we obtain

$$x = \frac{100 \times 65 \times 1}{125} = 52\%. \text{ Ans.}$$

(40) $AlCl_3$ or Al_2Cl_6.

(41) See Art. **8.**

(42) The name cæsium has been derived from the Latin *cæsius* (sky blue), and has reference to the two bright blue lines which distinguish the spectrum of this metal.

(43) See Art. **151.**

(44) Yes; see Art. **32.**

(45) Strontianite has the formula $SrCO_3$, and, therefore, contains 32.61 per cent. of oxygen. Witherite has the formula $BaCO_3$, and contains 24.39 per cent. of oxygen. Strontianite, therefore, contains 8.22 per cent. more oxygen than witherite.

(46) (*a*) 900 parts of gold,
100 parts of copper.
(*b*) 90 to 95 parts of copper,
10 to 5 parts of aluminum.
(*c*) 100 parts of tin,
8 parts of antimony,
1 part of bismuth,
4 parts of copper.

(47) See Art. **26.**

(48) See Art. **68.**

(49)
$$\underset{138}{K_2CO_3} + \underset{74}{Ca(OH)_2} = \underset{100}{CaCO_3} + \underset{112}{2KOH}$$

If 138 Kg. of K_2CO_3 yield 112 Kg. of KOH, 69 Kg. of K_2CO_3 will yield 56 Kg. of KOH. Ans.

(50) See Art. **149.**

(51) Calcium hydrate is obtained according to the equation:
$$\underset{56}{CaO} + \underset{18}{H_2O} = \underset{74}{Ca(OH)_2}$$

Using formula **10,** Art. **85,** *Theoretical Chemistry,*
$$w = \frac{Wm}{M},$$
and substituting the proper values, we obtain
$$w = \frac{250 \times 74}{56} = 330.357 \text{ Kg. Ans.}$$

(52) Rubidium was discovered by *Bunsen* in 1860 in the waters of the Durckheim Springs.

(53) See Art. **11.**

(54) Aluminum oxide occurs in nature as the minerals corundum, ruby, and sapphire. See Art. **169.**

(55) (*a*) and (*b*) See Art. **43.**

(56) NH_4NO_3.

(57) $5MgCO_3,2Mg(OH)_2,7Aq$.

(58) All metallic sulphates are soluble in water, except those of barium, strontium, and lead, which are almost entirely insoluble, and those of silver and calcium, as well as mercurous sulphate, which are only slightly soluble in water. See Art. **35.**

(59) (*a*) and (*b*) See Art. **106.**

(60) Common salt has the formula $NaCl$, and its molecular weight is 58.5. Using formula **1,** Art. **81,** *Theoretical Chemistry*, we obtain

$$\text{sodium} = \frac{100 \times 23 \times 1}{58.5} = 39.32\%;$$

$$\text{chlorine} = \frac{100 \times 35.5 \times 1}{58.5} = 60.68\%. \quad \textbf{Ans.}$$

(61) See Art. **22.**

(62) See Arts. **32** and **33.**

(63) See Art. **43.**

(64) $MgSO_4,H_2O,6Aq$.

(65) See Art. **127.**

(66) The formula of borax is $Na_2B_4O_7$, and its molecular weight is 202. Using formula **1,** Art. **81,** *Theoretical Chemistry,*

$$x = \frac{100\,a\,n}{m},$$

we obtain

$$\text{sodium} = \frac{100 \times 23 \times 2}{202} = 22.772\%;$$

$$\text{boron} = \frac{100 \times 11 \times 4}{202} = 21.782\%;$$

$$\text{oxygen} = \frac{100 \times 16 \times 7}{202} = 55.446\%. \quad \text{Ans.}$$

(67) Silicic oxide has the formula SiO_2 and a molecular weight of 60. Aluminum oxide has the formula Al_2O_3, and a molecular weight of 103. Employing formula **2,** Art. **82,** *Theoretical Chemistry,*

$$n = \frac{m\,x}{100\,a},$$

we obtain
$$n = \frac{163 \times 36.8}{100 \times 60} = 1,$$

and
$$n = \frac{163 \times 63.2}{100 \times 103} = 1;$$

or, in other words, the mineral fibrolite contains 1 molecule of silicic oxide and 1 molecule of aluminum oxide, hence its formula is SiO_2, Al_2O_3. Ans.

(68) See Art. **12.**

(69) See Art. **27.**

(70) Symbol Ca. Atomic weight 39.9. Valence II.

(71) Using formula **2,** Art. **82,** *Theoretical Chemistry,*

$$n = \frac{m\,x}{100\,a},$$

and substituting the proper values, we obtain

$$\text{magnesium} = \frac{246 \times 9.76}{24 \times 100} = 1;$$

$$\text{sulphur} = \frac{246 \times 13.01}{32 \times 100} = 1;$$

$$\text{oxygen} = \frac{246 \times 26.01}{16 \times 100} = 4;$$

$$\text{water} = \frac{246 \times 51.22}{18 \times 100} = 7.$$

Hence, the formula of this salt must be $MgSO_4, 7H_2O$, and its name is Epsom salt.

(72) See Art. **55.**

(73) Potassium. See Art. **18.**

(74) See Art. **36.**

(75) See Art. **42.**

(76) Cæsium was discovered simultaneously with rubidium by *Bunsen* in 1860 (see Art. **100**), and was obtained in the pure state by *Setterberg* in 1882. See Art. **102.**

(77) See Art. **169.**

(78) See Arts. **23** and **24.**

(79) BaO_2.

(80) Potassium chlorate is formed (see Art. **57**) according to the equation:

$$\underset{213}{3Cl_2} + \underset{336}{6KOH} = \underset{122.5}{KClO_3} + \underset{372.5}{5KCl} + \underset{54}{3H_2O}$$

Using formula **8,** Art. **85,** *Theoretical Chemistry,* and substituting the proper values, we obtain

$$(a)\quad W = \frac{35.5 \times 6 \times 336}{122.5} = 584.23 \text{ Kg. Ans.}$$

$$(b)\quad W = \frac{35.5 \times 6 \times 200}{122.5} = 347.76 \text{ g. Ans.}$$

(81) See Art. **100.**

(82) See Art. **131.**

(83) See Art. **87.**

(84) The alkaline sulphides, and those of calcium, barium, and strontium, are soluble in water.

(85) See Arts. **159-164.**

(86) See Art. **19.**

(87) See Art. **45.**

(88) Barium chlorate has the formula $Ba(ClO_3)_2$, and a molecular weight of 304. Using formula **1,** Art. **81,** *Theoretical Chemistry,*

$$x = \frac{100\,a\,n}{m},$$

and substituting the proper values, we obtain

$$\text{barium, } x = \frac{137 \times 100}{304} = 45.07\%;$$

$$\text{chlorine, } x = \frac{35.5 \times 2 \times 100}{304} = 23.35\%;$$

$$\text{oxygen, } x = \frac{16 \times 6 \times 100}{304} = 31.58\%. \text{ Ans.}$$

(89) (*a*) and (*b*) See Art. **48.**

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(1) $HNbO_3$.

(2) See Art. **214.**

(3) See Art. **2.**

(4) Niccolite has the formula $NiAs$; its molecular weight, therefore, is 133.5. Using formula **1,** Art. **81,** *Theoretical Chemistry,*

$$x = \frac{100\,a\,n}{m},$$

we find nickel $= x = \frac{100 \times 58.6 \times 1}{133.5} = 43.90\%$. Ans.

(5) Stannous chloride has the formula $SnCl_2$ and a molecular weight of 189. Using formula **1,** Art. **81,** *Theoretical Chemistry,*

$$x = \frac{100\,a\,n}{m},$$

we find that stannous chloride contains 37.56% of chlorine, or .3756 gram of chlorine in 1 gram of stannous chloride. 3.17 grams of chlorine occupy 1 liter; hence, .3756 gram of chlorine occupies the volume of .1185 liter, or 118.5 cubic centimeters.

(6) See Art. **160.**

(7) See Art. **191.**

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(8) Cobaltous nitrate is formed according to the equation:

$$\underset{74.5}{CoO} + \underset{126}{2HNO_3} = \underset{182.5}{Co(NO_3)_2} + \underset{18}{H_2O}$$

Using formula **6,** Art. **84,** *Theoretical Chemistry,*

$$z = \frac{m \times y}{a\,n},$$

and substituting the proper values, we obtain

$$z = \frac{74.5 \times 36}{182.5} = 14.69 \text{ g. Ans.}$$

(9) Chrome alum is formed according to the equation:

$$\underset{295}{K_2Cr_2O_7} + \underset{98}{H_2SO_4} + \underset{192}{3SO_2} = \underset{567}{2KCr(SO_4)_2} + \underset{18}{H_2O}$$

(*a*) Using formula **6,** Art. **84,** *Theoretical Chemistry,* and substituting the proper values, we obtain

$$K_2Cr_2O_7 = z = \frac{295 \times 998}{567} = 519.24 \text{ g. Ans.}$$

$$H_2SO_4 = z = \frac{98 \times 998}{567} = 172.5 \text{ g. Ans.}$$

(*b*) The solution of this question may be easily derived by simple proportion; that is,

$$567 : 18 = 998 : x = 31.682 \text{ g. } H_2O. \text{ Ans.}$$

(10) See Art. **177.**

(11) Fe_2O_3.

(12) Silver nitrate has the formula $AgNO_3$ and a molecular weight of 170. Using formula **1,** Art. **81,** *Theoretical Chemistry,*

$$x = \frac{100\,a\,n}{m},$$

and substituting the proper values, we obtain

$$x = \frac{100 \times 108 \times 1}{170} = 63.53\% \text{ silver. Ans.}$$

(13) Co_3O_4 or CoO, Co_2O_3.

(14) See Art. **206.**

(15) See Art. **32.**

(16) $GeCl_4$.

(17) (*a*) K_2CrO_4.
(*b*) $PbCrO_4$.
(*c*) K_2CrO_4, CrO_3 or $K_2Cr_2O_7$.

(18) See Art. **168.**

(19) See Art. **16.**

(20) See Arts. **86** and **87.**

(21) See Art. **146.**

(22)
$$\underset{295}{K_2Cr_2O_7} + \underset{24}{C_2} = \underset{153}{Cr_2O_3} + \underset{138}{K_2CO_3} + \underset{28}{CO}$$

Using formula **5,** Art. **84,** *Theoretical Chemistry,*

$$y = \frac{a\,n \times z}{m},$$

and substituting the proper values, we obtain

$$y = \frac{153 \times 853}{295} = 442.4 \text{ g. Ans.}$$

(23) Red hematite has the formula Fe_2O_3.
Iron pyrites has the formula FeS_2.
Magnetic iron ore has the formula Fe_3O_4.

Using formula **1,** Art. **81,** *Theoretical Chemistry,*

$$x = \frac{100\,a\,n}{m},$$

and substituting the proper values, we obtain

(*a*) red hematite $= x = \dfrac{100 \times 56 \times 2}{160} = 70\%$. Ans.

(*b*) iron pyrites $= x = \dfrac{100 \times 56 \times 1}{120} = 46.66\%$. Ans.

(*c*) magnetic iron ore $= x = \dfrac{100 \times 56 \times 3}{232} = 72.41\%$. Ans.

(24) Malachite has the formula $CuCO_3, Cu(OH)_2$ and a molecular weight of 220; hence, by proportion,

$$220 : 126 = 1{,}000 : x,$$

we obtain the weight of copper contained in 1,000 kilograms of malachite, i. e., 572.73 Kg.

(25) $PtCl_4$ has a molecular weight of 339. The formula then shows, that in 339 grams of $PtCl_4$, there are 142 grams of chlorine combined with 197 grams of Pt; and in 1 gram of $PtCl_4$, there is .419 gram of chlorine gas. According to *Inorganic Chemistry*, Part 1, 3.17 grams of chlorine gas occupy the volume of 1 liter, or 1,000 cubic centimeters, and, consequently, .419 gram of chlorine will occupy 132.176 cubic centimeters.

(26) See Art. **114.**

(27) See Art. **59.**

(28) Stannic chloride may be prepared by the direct union of chlorine and tin (see Art. **118**). The union may be expressed by the equation:

$$\underset{118}{Sn} + \underset{142}{Cl_4} = \underset{260}{SnCl_4}$$

which shows that 142 grams of chlorine are required to change 118 grams of tin into stannic chloride.

Chlorine gas is prepared according to the equation (see Art. **45,** *Inorganic Chemistry*, Part 1):

$$\underset{146}{4HCl} + \underset{87}{MnO_2} = \underset{126}{MnCl_2} + \underset{36}{2H_2O} + \underset{71}{Cl_2}$$

Using formula **6,** Art. **84,** *Theoretical Chemistry,*

$$z = \frac{m \times y}{a\,n},$$

and substituting the proper values, we obtain

$$HCl = z = \frac{146 \times 142}{71} = 292 \text{ g. Ans.}$$

$$MnO_2 = z = \frac{87 \times 142}{71} = 174 \text{ g. Ans.}$$

(29) Bi, Sb, Ta, Nb, and V. See also Art. **113.**

(30) $CuSO_4, 5H_2O$.

(31) Let x represent the percentage of Au.
Let y represent the percentage of Pt.

The Sp. Gr. of gold is 19.33, and of platinum is 21.5.

$$x+y = 100\%. \qquad (1)$$

$$19.33x+21.5y = 20\times 100 = 2{,}000. \qquad (2)$$

Multiplying (1) by 19.33, and subtracting from (2), we have

$$\begin{aligned} 19.33x+21.5y &= 2{,}000 \\ 19.33x+19.33y &= 1{,}933 \\ \hline 2.17y &= 67 \\ y &= 30.87\%.\ Pt. \quad \text{Ans.} \end{aligned}$$

(32) NH_4VO_3.

(33) $Co_3(AsO_4)_2, 8Aq$.

(34) See Art. **134.**

(35) See Art. **75.**

(36) See Art. **19.**

(37) $HgCl_2$ is formed according to the equation:

$$\underset{296}{HgSO_4}+\underset{117}{2NaCl} = \underset{142}{Na_2SO_4}+\underset{271}{HgCl_2}$$

Using formula **6,** Art. **84,** *Theoretical Chemistry,*

$$z = \frac{m\times y}{a\,n},$$

and substituting the proper values, we obtain

$$HgSO_4 = z = \frac{296\times 500}{271} = 546.1 \text{ g.} \quad \text{Ans.}$$

$$NaCl = z = \frac{117\times 500}{271} = 215.9 \text{ g.} \quad \text{Ans.}$$

(38) See Art. **30.**

(39) Uranium belongs to the chromium family of metals (see Art. **91**).

(40) See Art. **124.**

(41) Manganous sulphide has the formula MnS; its molecular weight is 87. Using formula **1,** Art. **81,** *Theoretical Chemistry,*

$$x = \frac{100\,a\,n}{m},$$

and substituting the proper values, we obtain

$$Mn = x = \frac{100 \times 55 \times 1}{87} = 63.22\%. \quad \text{Ans.}$$

$$S = x = \frac{100 \times 32 \times 1}{87} = 36.78\%. \quad \text{Ans.}$$

(42) See Art. **2.**

(43) MoS_2.

(44) Ta_2O_4 and Ta_2O_5.

(45) (*a*) Hg_2SO_4.
(*b*) $HgSO_4$.

(46) Stannic chloride is prepared according to the equation:

$$\underset{118}{Sn} + \underset{542}{2HgCl_2} = \underset{260}{SnCl_4} + \underset{400}{2Hg}$$

Using formula **6,** Art. **84,** *Theoretical Chemistry,*

$$z = \frac{m \times y}{a\,n},$$

and substituting the proper values, we obtain

$$z = \frac{118 \times 962}{260} = 436.6 \text{ g. tin.} \quad \text{Ans.}$$

(47) See Art. **80.**

(48) (*a*) Ferric chloride ($FeCl_3$). See Art. **14.**

(*b*) Ferric chloride is formed according to the equation:

$$\underset{56}{Fe} + \underset{106.5}{3Cl} = \underset{162.5}{FeCl_3}$$

Using formula **5,** Art. **84,** *Theoretical Chemistry,*

$$y = \frac{an \times z}{m},$$

and substituting the proper values, we obtain

$$y = \frac{162.5 \times 250}{56} = 725.45 \text{ g. } FeCl_3. \quad \text{Ans.}$$

(49) (*a*) Thallic hydroxide.

(*b*) Thallic sulphide.

(*c*) Thallious chloride.

(50) $MoCl_5$.

(51) WO_3.

(52) Antimonious sulphide has the formula Sb_2S_3, and a molecular weight of 336.

Using formula **1,** Art. **81,** *Theoretical Chemistry,*

$$x = \frac{100an}{m},$$

and substituting the proper values, we obtain

$$x = \frac{100 \times 120 \times 2}{336} = 71.43\% \ Sb. \quad \text{Ans.}$$

(53) See Art. **59.**

(54) See Art. **197.**

(55)
$$\underset{477}{2PbS} + \underset{96}{3O_2} = \underset{445}{2PbO} + \underset{128}{2SO_2}$$

Using formula **5,** Art. **84,** *Theoretical Chemistry,*

$$y = \frac{an \times z}{m},$$

and substituting the proper values, we obtain

$$y = \frac{128 \times 1{,}000}{477} = 268.34 \text{ Kg. } SO_2.$$

As 2.86 grams occupy the volume of 1 liter, 1 gram occupies the volume of .349 liter, and 268.34 kilograms occupy the volume of 93,650.66 liters.

(56) Hg_2O has a molecular weight of 416. Using formula **1,** Art. **81,** *Theoretical Chemistry,*

$$x = \frac{100\,a\,n}{m},$$

and substituting the proper values, we obtain

$$x = \frac{100 \times 400}{416} = 96.15\%\ Hg.$$

$$x = \frac{100 \times 16}{416} = 3.85\%\ O.$$

(57) See Art. **196.**

(58) See Art. **106.**

(59) Na_4ZrO_4.

(60) See Art. **198.**

(61) See Art. **198.**

(62) Silver nitrate may be formed according to the equation:

$$\underset{108}{Ag} + \underset{63}{HNO_3} = \underset{170}{AgNO_3} + \underset{1}{H}$$

Using formula **6,** Art. **84,** *Theoretical Chemistry,*

$$z = \frac{m \times y}{a\,n},$$

and substituting the proper values, we obtain

$$z = \frac{108 \times 250}{170} = 158.82 \text{ g. Ans.}$$

(63) RhO, RhO_2, RhO_3, and Rh_2O_3.

(64) See Art. **129.** Ce_2O_3 and CeO_2.

(65) See Art. **130.**

(66) MnO_2, manganese dioxide.
Mn_3O_4, red oxide of manganese.
MnO, manganous oxide.
Mn_2O_3, manganic oxide.
MnO_3, manganic anhydride.
Mn_2O_7, permanganic anhydride.

(67) (*a*) and (*b*) See Art. **178.**

(68) (1) $2PbS+3O_2 = 2PbO+2SO_2$.

(2) $PbS+2O_2 = PbSO_4$.

(3) $PbS+2PbO = 3Pb+SO_2$.

(4) $PbS+PbSO_4 = 2Pb+2SO_2$.

(69)
$$\underset{174}{2MnO_2}+\underset{48}{3O}+\underset{94}{K_2O} = \underset{316}{K_2Mn_2O_8}$$

Using formula **5,** Art. **84,** *Theoretical Chemistry,*

$$y = \frac{an \times z}{m},$$

and substituting the proper values, we obtain

$$y = \frac{316 \times 1{,}010}{174} = 1{,}834.25 \text{ g.} \quad \text{Ans.}$$

(70) See Art. **181.**

(71) See Art. **124.**

(72) Au_2O, AuO, and Au_2O_3.

(73) The specific gravity of platinum is 21.5 and that of nickel 8.6. From this data we obtain the proportion

$$21.5 : 8.6 = 392 : x,$$

or

$$21.5 : 8.6 = 392 : 156.8,$$

from which it follows that a vessel of the same shape and size, made of nickel, would weigh 156.8 grams.

(74) See Art. **38.**

(75) In chromic oxide Cr_2O_3, chromium is a triad.
In chromium dioxide CrO_2, chromium is a tetrad.
In chromium trioxide CrO_3, chromium is a hexad.
In chromous oxide CrO, chromium is a dyad.

QUALITATIVE ANALYSIS.

(PART 1.)

(1) See Art. **1.**

(2) (*a*) Chromium; (*b*) cobalt; (*c*) manganese. See also Arts. **55, 59,** and **66.**

(3) Aqua regia is a mixture of concentrate nitric and hydrochloric acids. See Art. **23,** 5.

(4) See Art. **2.**

(5) $AgNO_3 + HCl = AgCl + HNO_3$.

(6) See Art. **101.**

(7) Silver, lead, and mercury in the mercurous condition.

(8) (*a*) and (*b*) See Art. **8.**

(9) (*a*) and (*b*) See Art. **8.**

(10) Strontium. See also Art. **73.**

(11) Silver, lead, mercurous, mercuric, and copper.

(12) See Art. **108.**

(13) Manganese. See Art. **67.**

(14) Ferrous, black; ferric, black; cobalt, black; nickel, black; chromium, green; manganese, flesh color; aluminum, white; zinc, white.

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(15) Lead, mercurous, barium, strontium, and calcium.

(16) Both metals are precipitated by sodium hydrate, and the precipitate is dissolved in an excess of the reagent. From this solution aluminum is reprecipitated by ammonium chloride, but not by hydrogen sulphide; while zinc is reprecipitated by hydrogen sulphide, but not by ammonium chloride. See Arts. **52,** 2 and **63,** 2.

Another good method of distinguishing between them is to add ammonium chloride and ammonium hydrate to the solution. Aluminum will thus be precipitated from this solution, while zinc will not.

(17) $BaCl_2 + H_2SO_4 = BaSO_4 + 2HCl.$

(18) See Art. **92.**

(19) (*a*) and (*b*) See Art. **4.**

(20) See Art. **94.**

(21) Heating with concentrate sulphuric acid. See Art. **127.**

(22) A sulphide. See Art. **4.**

(23) Aluminum and chromium.

(24) See Art. **146,** 2.

(25) See Art. **93.**

(26) Antimony, stannous, stannic, ferric, and aluminum.

(27) Heating with sodium hydrate. See Art. **81,** 3.

(28) Bismuth and antimony.

(29) Cadmium, arsenious, arsenic, and stannic. Sometimes the sulphide of antimony has a yellowish color.

(30) See Art. **94.**

(31) See Art. **145.**

(32) See Arts. **58,** 6 and **104.**

(33) (*a*) Red.

(*b*) $2SbCl_3 + 3H_2S = Sb_2S_3 + 6HCl$.

(34) This indicates that the compound is a salt of one of the acids of sulphur.

(35) If a solution of calcium sulphate is added, barium, if present, will be precipitated at once, strontium after a little time, and calcium will not be precipitated at all. The solution may be made very dilute, and sulphuric acid used, instead of calcium sulphate, with the same results. The colors imparted to the flame serve well to distinguish between them; or, they may be identified by the method used to separate them. See Art. **105.**

(36) See Art. **146,** 8.

(37) Only manganese and zinc or one of these metals can be present.

(38)

$$\underset{\substack{\text{magnesium}\\ \text{sulphate}}}{MgSO_4} + \underset{\substack{\text{sodium}\\ \text{phosphate}}}{Na_2HPO_4} + \underset{\substack{\text{ammonium}\\ \text{hydrate}}}{NH_4OH}$$

$$= \underset{\substack{\text{magnesium}\\ \text{ammonium}\\ \text{phosphate}}}{MgNH_4PO_4} + \underset{\substack{\text{sodium}\\ \text{sulphate}}}{Na_2SO_4} + \underset{\text{water}}{H_2O}$$

(39) (*a*) Ag_2O, brown.

(*b*) $Pb(OH)_2$, white.

(*c*) Hg_2O, black.

(*d*) HgO, yellow.

(*e*) $Cu(OH)_2$, blue.

(40) Yellowish green.

(41) See Art. **25,** 5. The intermediate colors may not be seen unless the reagent is added carefully, but the white and the black precipitates are always seen.

(42) Oxalic acid. See Art. **126,** 4.

(43) From acid solutions, copper is precipitated by hydrogen sulphide, while nickel is not.

(44) See Arts. **118,** 1 and **146,** 11.

(45)

$$\underset{\substack{\text{sodium}\\\text{arsenite}}}{2Na_2HAsO_3} + \underset{\substack{\text{hydrogen}\\\text{sulphide}}}{3H_2S} + \underset{\substack{\text{hydro-}\\\text{chloric}\\\text{acid}}}{4HCl}$$

$$= \underset{\substack{\text{arsenious}\\\text{sulphide}}}{As_2S_3} + \underset{\substack{\text{sodium}\\\text{chloride}}}{4NaCl} + \underset{\text{water}}{6H_2O}$$

(46) Aluminum and zinc. Manganese gives a light-colored precipitate, but it is not white.

(47) By the addition of an oxidizing agent, such as nitric acid, bromine water, a permanganate, etc.

(48) To hold the metals of the succeeding groups in solution.

(49) The alkalies and alkaline earths.

(50) Mercury, arsenic, tin, and iron. In one sense chromium also forms two series of salts, as it acts both as a base and an acid, forming chromium salts and chromates. The same may be said of manganese.

(51) (*a*) The sharp, disagreeable odor of acetic acid.

(*b*) The pleasant odor of acetic ether.

(52) Lead and mercury.

(53) (*a*) Yellow arsenious sulphide As_2S_3.

(*b*) Ammonia, ammonium sulphide, ammonium carbonate, and hot concentrate nitric or sulphuric acid, but is insoluble in hydrochloric acid.

(54) $Hg_2(NO_3)_2 + 4H_2SO_3 = Hg_2 + 4H_2SO_4 + 2NO.$

(55) It fuses easily to a bright, metallic globule, giving the flame a pale, bluish tinge, and deposits a yellow volatile incrustation on the charcoal.

(56) Cadmium.

(57) See Arts. **111** and **146,** 1.

(58) Arsenic, or one of its compounds.

(59) (*a*) White barium sulphate $BaSO_4$.
(*b*) White barium thiosulphate BaS_2O_3.
(*c*) White barium sulphite $BaSO_3$.

(60) When treated with hydrochloric acid, barium sulphate is not attacked; barium thiosulphate dissolves, giving off sulphur dioxide and throwing out free sulphur, which gives the liquid a yellowish color; and barium sulphite dissolves with the evolution of sulphur dioxide.

(61) Chromic acid and chromates are generally reduced by heating with hydrochloric acid and alcohol, or with sulphurous acid; but other reducing agents may be used.

(62) (*a*) Black mercuric sulphide HgS, together with some free mercury.

(*b*) It dissolves slowly in hot concentrate hydrochloric acid, and readily in aqua regia.

(63) See Art. **9.**

(64)
(65)
(66)
(67)

These solutions should be tested for single metals. They are intended to test the student's knowledge of the section on the "Deportment of the Metals with Reagents."

(68)
(69)
(70)

These solutions should be treated as directed for the "Analysis of Mixed Solutions." No. 69 does not contain members of Division B of Group II, and No. 70 does not contain members of Division A of Group II.

(71)
(72)
(73)
(74)
(75)

Each of these compounds contains one metal and one acid. The metal should be determined by making use of the reactions described under "Deportment of the Metals with Reagents," and the acid by means of the reactions described for the common acids.

QUALITATIVE ANALYSIS.

(PART 2.)

(1) See Art. **1.**

(2) (*a*) A compound of tin, aluminum, zinc, barium, strontium, calcium, magnesium, silica, or possibly a silicate.

(*b*) A drop or two of cobalt nitrate should be added, and the mass should be again heated in the oxidizing blowpipe flame to the highest temperature obtainable. See also Art. **18.**

(3) Magnesium oxide, or a compound of magnesium that has been reduced to the oxide on the charcoal. See also Art. **18.**

(4) (*a*) and (*b*) See Art. **75.**

(5) (*a*) The pungent odor of acetic acid.

(*b*) A little of the substance should next be heated with concentrate sulphuric acid and alcohol, when an acetate if present will be recognized by the agreeable odor of acetic ether. See also Art. **24.**

(6) The alkaloids are first divided into volatile and non-volatile alkaloids. The non-volatile alkaloids are divided into three groups, as follows:

Group I consists of those alkaloids that are precipitated from solutions of their salts by sodium hydrate, and dissolve in an excess of the reagent. The common ones are morphine and cocaine.

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Group II consists of those alkaloids that are precipitated by sodium hydrate, and are insoluble in an excess of the reagent; and are also precipitated by sodium bicarbonate, even from acid solutions. The common ones are quinine, cinchonine, and narcotine.

Group III consists of those alkaloids that are precipitated by sodium hydrate, and are insoluble in an excess of the reagent, but are not precipitated from acid solutions by sodium bicarbonate. The most common ones are strychnine, brucine, and atropine.

(7) In the form of powder or small crystals.

(8) (*a*) and (*b*) See Art. **13.**

(9) A salt of one of the acids of sulphur.

(10) See Art. **29.**

(11) Earthy and alkaline phosphates. See also Art. **82.**

(12) Nicotine and conine.

(13) See Art. **2.**

(14) (*a*) A nitrate or chlorate.
(*b*) A chlorate. See also Art. **12.**

(15) If the substance decrepitates, deflagrates, fuses, or volatilizes; if a metallic globule forms with an incrustation, if a metallic globule forms without an incrustation, if an incrustation is formed without a metallic globule, if a white luminous mass is formed, if a colored mass is formed.

(16) Manganese and chromium. See also Art. **23.**

(17) (*a*) A carbonate.

(*b*) The gas should be tested at the mouth of the tube with a drop of barium hydrate, and then a fresh sample should be treated with hydrochloric acid, and the escaping gas again tested with barium hydrate. See also Art. **24.**

(18) Its oxide, thoria, is the chief constituent used in the mantle of the Welsbach light.

(19) (*a*) and (*b*) See Arts. **60** and **61.**

(20) A cyanide that is decomposed by heat.

(21) An ammonium compound, or possibly a nitrogenous organic compound or a cyanide containing water. See also Art. **4.**

(22) (*a*) A compound of arsenic.

(*b*) Sulphide of arsenic is indicated.

(23) The substance is an oxalate, and is decomposed by the sulphuric acid, with the evolution of carbon monoxide and carbon dioxide.

(24) See Art. **25.**

(25) Lithium, cæsium, and rubidium.

(26) See Art. **72.**

(27) (*a*) It should be slightly acid.

(*b*) By means of two pieces of litmus paper, one of which is faintly red, and the other faintly blue. See also Art. **76.**

(28) See Art. **108.**

(29) (*a*) Some organic compounds and ferrocyanides.

(*b*) It may be recognized by the blue flame with which it burns, when ignited at the mouth of the tube.

(30) Gold, platinum, iridium, molybdenum, selenium, and tellurium.

(31) Lead, copper, and zinc. See also Art. **72.**

(32) See Art. **65.**

(33) Quinine, cinchonine, and narcotine.

(34) Zinc oxide, or a compound which is reduced to zinc oxide by heat. See also Art. **5.**

(35) A compound of lead.

(36) A compound of barium, copper, thallium, molybdenum, or possibly a borate or phosphate. See also Art. **21.**

(37) See Art. **40,** 6.

(38) By evaporating a sample of the water to dryness, and igniting the residue; when, if organic matter is present, it will char, and generally give off a burnt odor. See also Art. **69.**

(39) See Art. **107.** Probably heating with sulphuric acid and potassium dichromate, or with chromic acid, is the most characteristic test.

(40) See Art. **77.**

(41) (*a*) By rendering turbid a drop of barium hydrate.
(*b*) A carbonate or oxalate.

(42) It readily fuses to a metallic globule, and white fumes of the oxide are given off. which form a white incrustation on the charcoal.

(43) The colors imparted to the flame by metals are due to highly heated luminous vapors; hence, the substance must be volatilized before the flame is colored.

(44) See Art. **100.**

(45) To determine the acid of the compound.

(46) This shows that the substance contains arsenic, probably in the form of oxide As_2O_3.

(47) (*a*) Green.
(*b*) Bright red.
(*c*) Brick red.

(48) That the substance is iodine or an iodide.

(49) Volatile compounds must be used; and, as chlorides are usually the most volatile, they are most suitable for this purpose.

(50) (*a*) and (*b*) See Art. **67.**

(51) The substance should be heated with concentrate nitric acid, which decomposes all bromides except silver bromide, setting free reddish-brown vapors of bromine, which condense in the upper part of the tube, forming red globules.

(52) (*a*) An organic substance.
(*b*) Tartaric acid or a tartrate.

(53) See Art. **22.**

(54) (*a*) See Art. **47.**

(*b*) Distinctive reactions may be obtained with the borax bead, and by adding zinc to an acid solution of vanadium.

(55) See Art. **68.**

(56) Upon the fact that sugar is a reducing agent, and reduces the cupric solution, forming red cuprous oxide Cu_2O.

(57) (*a*) and (*b*) See Art. **60.**

(58) Tartaric acid or a tartrate.

(59) See Art. **79.**

(60) See Art. **57.**

(61) See Arts. **88, 89,** and **90.**

(62) Morphine, cocaine, quinine, cinchonine, narcotine, strychnine, brucine, and atropine.

(63) (*a*) and (*b*) See Art. **81.**

(64) (*a*) and (*b*) See Art. **106.**

(65) Thallium and tungsten.

(66) (*a*) and (*b*) See Art. **77.**

(67) (*a*) and (*b*) See Art. **46.**

(68) The oxidizing agent would convert the phosphorus into phosphoric acid; but, as phosphoric acid is a normal constituent of nearly all animal and vegetable bodies, it would be impossible to say whether the phosphoric acid thus found had come from oxidizing free phosphorus or not.

NOTE.—The substances given for analysis are omitted from the Key, but the student will be told whether his analyses are correct or not.

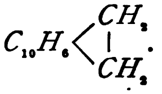
ORGANIC CHEMISTRY.

(PART 1.)

(1) The percentage figures, 40, 6.6, and 53.4, divided by their atomic weights give $\frac{40}{12} = 3.3$, $\frac{6.6}{1} = 6.6$, and $\frac{53.4}{16} = 3.3$. If this ratio, 3.3 : 6.6 : 3.3, is expressed in its lowest term, it becomes 1 : 2 : 1, and the empirical formula of the compound is CH_2O. See Art. **59.**

(2) See Art. **49.**

(3) See Art. **77.**

(4) 

$C_{10}H_6\left\langle\begin{matrix}CH_2\\|\\CH_2\end{matrix}\right.$.

(5) (*a*) $11m + 5.5n + 7.8p$.

(*b*) See Art. **38.**

(6) Equal volumes of different solutions, at the same temperature and osmotic pressure, contain equal numbers of molecules of dissolved substances. See also Art. **51.**

(7) See Art. **76.**

(8) See Art. **26.**

(9) See Art. **96.**

(10) See Art. **70.**

(11) See Art. **68.**

(12) See Art. **41.**

(13) $x = 499 \times \frac{273 + 9.87}{273 + 29.49} = 466.615$ mm. pressure of mercury at 9.87°. See also Art. **51.**

(14) See Art. **59.**

(15) C_nH_{2n-12}.

(16) See Art. **79.**

(17) See Art. **64.**

(18) See Art. **13.**

(19) See Art. **47.**

(20) $x = 536 \times \frac{273 + 16.25}{273 + 30} = 511.666$ mm. pressure at 16.25°. See Art. **51** for explanations.

(21) See Art. **61.**

(22) Butine belongs to the acetylene series of hydrocarbons.

(23) See Art. **51.** Solutions which, under equal conditions, exert the same osmotic pressure, are said to be isotonic.

(24) See Art. **71.**

(25) See Art. **62.**

(26) See Art. **4.**

(27) See Art. **33.**

(28) See Art. **68** and Art. **104** of *Inorganic Chemistry*, Part 2.

(29) The percentage figures, 56, 12, 32, divided by their atomic weights give $\frac{56}{12} = 4.66$, $\frac{12}{1} = 12$, and $\frac{32}{16} = 2$, or 2.33 : 6 : 1. In order to obtain fractionless figures, we multiply by 3, and obtain 7 : 18 : 3, from which we take the empirical formula of the compound to be $C_7H_{18}O_3$.

(30) The mineral product known as petroleum, rock oil, or naphtha.

(31) Hydrocarbons are composed of carbon and hydrogen atoms in various groupings.

(32) See Art. **51.**

(33) See Art. **38.**

(34) See Art. **8.**

(35) $CH_3 \cdot C \vdots CH$.

(36) See Art. **53.**

(37) Pentane C_5H_{12}, boiling point 36°.
Octane C_8H_{18}, boiling point 125°.
Hexadecane $C_{16}H_{34}$, boiling point 287°.

(38) C_nH_{2n+2}, or $H_3C \cdot C_nH_{2n} \cdot CH_3$.

(39) See Art. **46.**

(40) See Arts. **21** and **23.**

(41) See Art. **58.**

(42) See Art. **88.**

(43) Hexane, $CH_3 \cdot CH_2 \cdot CH_2 \cdot CH_2 \cdot CH_2 \cdot CH_3$.
Nonane, $CH_3 \cdot CH_2 \cdot CH_2 \cdot CH_2 \cdot CH_2 \cdot CH_2 \cdot CH_2 \cdot CH_2 \cdot CH_3$.
Butane, $CH_3 \cdot CH_2 \cdot CH_2 \cdot CH_3$.

(44) Alcohols are compounds of C, O, and H. The alcohol molecule is constructed upon the model of the water molecule, in which half the H is replaced by a compound radical, which generally consists of C and H.

(45) Using the formula given in Art. **49,**

$$M = \frac{rp \times 100}{dg},$$

and substituting the known values, we obtain

$$M = \frac{18.9 \times 4 \times 100}{1.148 \times 111} = 59.33,$$

a number sufficiently close to confirm the otherwise obtained molecular weight (60) of urea.

(46) See Art. **37.**

(47) See Arts. **11** and **12.**

(48) C_nH_{2n-6} (n must not be smaller than 6).

(49) See Art. **64.**

(50) See Arts. **71** and **72.**

(51) See Art. **40.**

(52) See Art. **18.**

(53) See Art. **102.**

(54) .3059 gr. gave .6 gr. CO_2 and .3040 gr. H_2O. Since 44 gr. CO_2 contain 12 gr. of C, $\frac{12}{44}$ of .6, or .1636 gr., is the weight of C found; and, since 18 gr. H_2O contain 2 gr. of H, $\frac{2}{18}$ of .3040 gr., or .0338, is the weight of H. The amount of O is the difference between the original weight .3059 and the sum of the weights of H and C. $.1636+.0338 = .1974$; namely, .1085 gr. of O. Then the original compound contained .1636 gr., or 53.48% of C; .0338 gr., or 11.05% of H; .1085 gr., or 35.47% of O.

(55) See Art. **22.**

(56) See Art. **91.**

(57) See Art. **71.**

(58) $CH_3{\cdot}CH{:}CH_2$.

(59) Carbohydrates are compounds which contain 6, or some multiple of 6, atoms of C, together with some multiple of the group H_2O.

(60) See Art. **70.**

(61) Using the formula given in Art. **49,**

$$M = \frac{rp \times 100}{dg},$$

and substituting the proper values, we obtain

$$M = \frac{18.9 \times 3 \times 100}{0.639 \times 106} = 83.71,$$

which is close enough to confirm the opinion that the molecular weight of lactic acid is 90, and that its formula is $C_3H_6O_3$.

(62) See Art. **20.**

(63) (1) $C_2H_5 \cdot H$; (2) $H_3C \cdot OH$; (3) $CH_3 \cdot CH_2I$.

(64) See Art. **73.**

(65) C_nH_{2n-2}.

(66) See Art. **10.**

(67) $CH_3 \cdot CH_2 \cdot CH_3$. For the graphical representation see Fig. 18.

(68) See Art. **81,** and Arts. **107, 108, 109,** and **110** of *Inorganic Chemistry*, Part 2.

(69) See Fig. 16, Art. **68.**

(70) See Arts. **102, 103,** and **104,** *Inorganic Chemistry*, Part 2, and Art. **73.**

(71) C_nH_{2n}.

(72) $H_3C \cdot CH_3$.

(73) Three; namely, normal pentane, secondary pentane or isopentane, and tertiary pentane or tetramethylmethane.

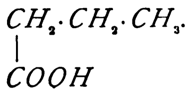
(74) See Art. **5.**

(75) See Art. **54.**

ORGANIC CHEMISTRY.

(PART 2.)

(1) Normal butyric acid has the formula $CH_2 \cdot CH_2 \cdot CH_3$. For its preparation see Art. **75.**



$$\underset{COOH}{CH_2} \cdot CH_2 \cdot CH_3.$$

(2) See Art. **7.**

(3) From the equations showing the synthetical preparation of citric acid given in Art. **157,** we see that 58 parts of acetone and 142 parts of chlorine are necessary to obtain 192 parts of citric acid; hence,

192 : 58 = 228.39 : 68.97 grams of acetone;
192 : 142 = 228.39 : 168.89 grams of chlorine.

As 1 liter of chlorine weighs 3.17 grams, 168.89 grams = 53.28 liters.

$$\left.\begin{array}{l}\text{68.97 grams acetone.}\\ \text{53.28 liters chlorine.}\end{array}\right\}\text{Ans.}$$

(4) Potassium xanthate is formed according to the equation:

$$\underset{46}{C_2H_5 \cdot OH} + \underset{76}{CS_2} + \underset{56}{KOH} = \underset{160}{C_2H_5 \cdot O \cdot CS_2K} + \underset{18}{H_2O}$$

Using formula **8,** *Theoretical Chemistry,*

$$W = \frac{Mw}{m}.$$

Substituting the known values, we obtain

$$W = \frac{46 \times 192}{160} = 55.2 \text{ grams. Ans.}$$

(5)

$CH_3\cdot OH$ methyl alcohol.	$CH_3\cdot CO_2H$ acetic acid.
$C_2H_5\cdot OH$ ethyl alcohol.	$C_2H_5\cdot CO_2H$ propionic acid.
$C_4H_9\cdot OH$ butyl alcohol.	$C_4H_9\cdot CO_2H$ valeric acid.
$C_8H_{17}\cdot OH$ octyl alcohol.	$C_8H_{17}\cdot CO_2H$ pelargonic acid.

(6) Compounds obtained by synthesis are optically inactive.

(7) See Art. **128.**

(8) Monochlor-acetic acid.

(9) Acetic acid $CH_3\cdot CO_2H$; carbon dioxide CO_2; and water H_2O.

(10) See Art. **11.**

(11) See Art. **128.**

(12) Sodium stearate.

(13) See Art. **5.**

(14) An optically active organic compound is one which possesses the property of turning the plane of polarization, and it is a compound which possesses one or more asymmetric carbon atoms.

(15) Citric acid has the formula $C_3H_4(OH)(CO_2H)_3$. Its adulteration with tartaric acid is detected by adding to a citric-acid solution some potassium-acetate solution, which precipitates the tartaric acid, if present, as white, crystalline cream of tartar.

(16) Salicylic aldehyde, or oil of meadowsweet.

(17)
$$\underset{90}{CO_2H\cdot CO_2H} = \underset{46}{H\cdot CO_2H} + \underset{44}{CO_2}$$

Using formula **8**, *Theoretical Chemistry*,

$$W = \frac{Mw}{m}.$$

Substituting the known values, we obtain

$$W = \frac{90\times 100}{46} = 195.65 \text{ kilos. Ans.}$$

(18) See Art. **109.**

(19) (*a*) $C_2H_5 \cdot OH$; (*b*) $CH_3 \cdot O \cdot CH_3$; (*c*) $(CH_3CO)_2O$; (*d*) $CH_3 \cdot CHO$.

(20) See Art. **111.**

(21) See Art. **158.**

(22) See Art. **35.**

(23) Diethyl sulphide $C_2H_5 \cdot S \cdot C_2H_5$; for its preparation see Art. **27.**

(24) See Arts. **57** and **102.**

(25) For the determination of calcium.

(26) See Art. **1.**

(27) See Art. **10.**

(28) Fehling's solution is an alkaline solution of cupric tartrate, and is used in analytical chemistry for the qualitative and quantitative determination of sugar.

(29) See Art. **64.**

(30) $C_6H_3(OH)_2 \cdot CO_2H$ is protocatechuic acid, and is a phenol acid belonging to the class of monobasic hydroxy acids obtained from the benzene hydrocarbons.

(31) See Arts. **13** and **14.**

(32) See Arts. **5** and **110.**

(33) By oxidation of the corresponding alcohol. See Art. **47.**

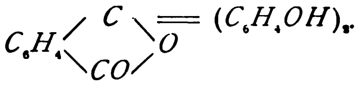
(34) See Art. **76.**

(35) Allyl alcohol belongs to the monohydric alcohols of the olefine series; its formula is $C_3H_5 \cdot OH$. For its preparation see Art. **21.**

(36) Yes; racemic is an isomeride of tartaric acid, it is externally compensated tartaric acid.

(37) (*a*) and (*b*) See Arts. **4** and **5.**

(38) Phenol phthalein has the formula



It is frequently used as an indicator in volumetric analysis. See *Theoretical Chemistry*, Art. **116,** and other places.

(39) See Art. **9.**

(40) $C_6H_3(OH)_2 \cdot CH:CH \cdot CO_2H$ is caffeic acid. For its preparation see Art. **125.**

(41) See Art. **36.**

(42) Potassium formate is obtained according to the equation :

$$\underset{28}{CO}+\underset{56}{KOH}=\underset{84}{HCO \cdot OK}$$

500 liters of CO = 625 grams, then

28 : 84 = 625 : 1,875 grams. Ans.

(43) See Art. **148.**

(44)

$$Ca(C_2H_5SO_4)_2+2KSH=CaSO_4+K_2SO_4+2(C_2H_5 \cdot SH).$$

(45) $C_nH_{2n\ 2}(CO_2H)_2$.

(46) By the action of heat and sulphuric acid, lactic acid is decomposed into aldehyde and formic acid, and if the sulphuric acid is of sufficient strength, this acid acts further on the newly formed formic acid, decomposing it into water and carbon monoxide.

(47) See Art. **5.**

(48) $C_nH_{2n}(COOH)_2$.

(49) Mannite or mannitol. See Art. **43.**

(50) Tartaric acid has the formula $C_2H_2(OH)_2(CO_2H)_2$. For its preparation see Art. **145.**

(51) See Art. **69.**

(52) $CH_2(CO_2H)_2$ is malonic acid. For its preparation see Art. **130.**

(53) A diatomic, or dihydric, alcohol is an alcohol containing two hydroxyl (HO) groups.

(54) Writing ink is a mixture of gallotannic acid and ferrous sulphate to which gum arabic has been added to give the fluid the proper adhesive and drying properties.

(55) $CH_3\cdot CHO$. For the preparation of acetic aldehyde see Art. **50.**

(56) See Art. **17.**

(57) $(C_6H_5CO)_2O$ is benzoic anhydride, or dibenzoyl oxide, and $(C_6H_5CO)_2O_2$ is benzoic peroxide. For their preparation see Art. **102.**

(58) Palmitic acid.

(59) See Art. **118.**

(60) (*a*) See Art. **23.** (*b*) Aromatic alcohols.

(61) See Art. **116.**

(62) See Art. **57.**

(63) See Arts. **35** and **108.**

(64) (*a*) Monohydric alcohols of the paraffin series are produced by the replacement of one hydrogen atom of a paraffin hydrocarbon by the group OH. (*b*) Their general formula is $C_nH_{2n+1}\cdot OH$.

(65) Fumaric acid exists in two modifications; namely,

$$CO_2H\cdot CH : CH\cdot CO_2H$$

and

$$CH_2 : C(CO_2H)_2$$

(66) See Art. **87.**

(67) All optically active compounds contain one or more carbon atoms, to which four different elements or radicals are attached.

(68) See Art. **163.**

(69) See Art. **6.**

(70) See Art. **68.**

(71) See Art. **39.**

(72) See Art. **136.**

(73) See Art. **158.**

(74) Hexa-hydroxy-anthraquinone $C_{14}H_2(OH)_6O_2$ is obtained as a red precipitate by heating 1 part of gallic acid $C_6H_2(OH)_3 \cdot CO_2H$ with 4 parts of H_2SO_4, and pouring the cooled mixture in water.

ORGANIC CHEMISTRY.

(PART 3.)

(1) By adding a little mercuric chloride, this compound destroys the red color of ferric thiocyanate, but does not change that of ferric acetate.

(2) See Art. **88.**

(3) $C_2H_5 \cdot OH + HNO_3 = C_2H_5 \cdot NO_3 + H_2O.$

(4) Potassium chloride and potassium formate, as is seen from the following equation:

$$CHCl_3 + 4KOH = 3KCl + HCO \cdot OK + 2H_2O$$

(5) See Art. **43.**

(6) The equation

$$Zn_2 + 2C_2H_5I = Zn(C_2H_5)_2 + ZnI_2$$

represents the formation of zinc ethide; taking the molecular weight of zinc at 130, and that of zinc ethide as 123, we have the proportion: $130:123 = x:194$, when $x = 205.04$ grams of zinc. Ans.

(7) A nitrile base is a tertiary amine compound, having the general formula NR''', or NR_3'.

(8) Amido-acetic acid.

(9) Preparation of potassium cyanide (see Arts. **131** and **133**):

$$4KCNFe(CN)_2 = 4KCN + FeC_2 + N_2$$

or

$$4KCN,Fe(CN)_2 + K_2CO_3 = 5KCN + KCNO + CO_2 + Fe$$

(10) See Art. **1.**

(11) See Art. **15.**

(12) See Art. **147.**

(13) $C_6H_5 \cdot NH \cdot NH_2$ is phenyl hydrazine; for its preparation see Art. **128.**

(14) See Art. **103.**

(15) See Art. **75.**

(16) The term "metallo-organic compounds" is applied to compounds composed of alcohol radicals and metallic atoms.

(17) $(C_2H_5)_2SO_4$ is ethyl sulphate.

(18) See Art. **147.**

(19) See Art. **125.**

(20) Carbon monoxide, carbon dioxide, ammonia, hydrocyanic acid, and urea, as is seen from the following equation:

$$2(CONH_2)_2 = CO + CO_2 + NH_3 + CNH + CN_2H_4O$$

(21) See Art. **74.**

(22) $C_6H_5NO_2 + 3H_2 = C_6H_5NH_2 + 2H_2O$. The chief product is aniline.

(23) See Art. **40.**

(24) Ammonium chloride and ammonium cyanide, as is seen from the equation:

$$CHCl_3 + 5NH_3 = 3NH_4Cl + NH_4CN$$

(25) See Art. **3.**

(26) Methyl cyanide has the formula CH_3CN, or $H_3\equiv C-C\equiv N$, nitrogen being trivalent.

(27) A substantive dye is a colored insoluble compound formed by the dyestuff and the fiber of the material to be dyed.

(28) See Art. **94.**

(29) See Art. **74.**

(30) See Art. **56.**

(31) See Art. **30.**

(32) Chloroform is distinguished by its anesthetic property.

(33) Prussian blue is ferric ferrocyanide; its formula is $Fe_4[Fe(CN)_6]_3$.

(34) See Art. **119.**

(35) See Art. **89.**

(36) All cacodyl compounds are extremely poisonous.

(37) See Art. **51.**

(38) See Art. **23.**

(39) Ethylene chloride or ethene chloride C_2H_4Cl.

(40) See Art. **132.**

(41) $C_6H_5\cdot NH_2HNO_3+HNO_2=C_6H_5\cdot N:N\cdot NO_3+2H_2O$.

(42) Benzyl amine has the formula $C_6H_5\cdot CH_2NH_2$, and toluidine has the formula $C_6H_4CH_3\cdot NH_2$, from which it is seen that both compounds have the same molecular weight and the same percentage composition.

(43) See Art. **74.**

(44) See Art. **53.**

(45) See Art. **27.**

(46) See Art. **14.**

(47) See Art. **131.**

(48) $CH_2(NH_2)\cdot CO_2H$.

(49) NH_2CH_3. See Art. **75** for the properties of methyl amine.

(50) See Art. **61.**

(51) See Art. **31.**

(52) See Art. **16.**

(53) Ethyl iodide is decomposed by the action of the sun into iodine and butane, according to the equation:

$$2C_2H_5I = C_4H_{10} + I_2$$

(54) Prussic acid has the formula $HC \vdots N$; for its preparation see Art. **132.**

(55) A diazo reaction takes place when nitrous oxides act on an ammonia base; the result is a diazo compound, containing the group $\cdot N : N \cdot$.

(56) See Art. **90.**

(57) Bunsen and Berzelius.

(58) Dynamite is an infusorial earth, known as kieselguhr, impregnated with nitroglycerin.

(59) Trichloracetic acid has the formula $CCl_3\cdot CO_2H$. It may be obtained by oxidizing chloral with nitric acid. See also Art. **23.**

(60) Ethylene chloride is obtained from glycol according to the following equation:

$$C_2H_4(OH)_2 + 2PCl_5 = C_2H_4Cl_2 + 2POCl_3 + 2HCl$$

Taking the molecular weight of ethylene chloride as 99, and that of glycol as 62, we have the proportion $99 : 62 = 29 : x$, when $x = 18.16$ grams. Ans.

(61) Turnbull's blue is ferrous ferricyanide; its formula is $Fe_3[Fe(CN)_6]_2$.

(62) See Art. **119.**

(63) See Art. **86.**

(64) $PH_2 \cdot C_6H_5$ is phenyl phosphine.

(65) See Art. **45.**

(66) $$\underset{}{CCl_3 \cdot COH} + NaOH = \underset{\text{sodium formate}}{NaCHO_2} + \underset{\text{chloroform}}{CHCl_3}$$

(67) An adjective dye is a colored insoluble compound, formed by the dyestuff with a mordant.

(68) $(CONH_2)_2$ is oxamide. See Art. **101.**

(69) $$CH_2ClCO_2K + KNO_2 + H_2O$$
$$= KCl + CH_3NO_2 + KHCO_3$$

(70) In two; namely, one having the general formula CH_2XCH_2, and another having the formula $CHXCH_3$.

(71) Tribromhydrin is prepared according to the following equation:

$$C_3H_5I + 2Br_2 = IBr + C_3H_5Br_3$$

Taking the molecular weight of allyl iodide as 167.5, that of bromine as 159.5, and that of tribromhydrin as 280.25, we have the following proportions:

$$167.5 : 280.25 = x : 250$$

when x is 149.42 grams of allyl iodide, and

$$319 : 280.25 = x : 250$$

when x is 284.567 grams of bromine. Ans.

ORGANIC CHEMISTRY.

(PART 4.)

(1) See Art. **25.**

(2) See Arts. **137–140.**

(3) See Arts. **158** and **159.**

(4) Albuminous bodies chiefly differ from other chemical compounds in that they neither crystallize nor volatilize.

(5) Dextrin and maltose are obtained from starch according to the equation:

$$\underset{\text{starch}}{3C_6H_{10}O_5} + H_2O = \underset{\text{maltose}}{C_{12}H_{22}O_{11}} + \underset{\text{dextrin}}{C_6H_{10}O_5}$$

The molecular weights are as follows: starch, 162; maltose, 342; and dextrin, 162. Using simple proportions, we obtain

$486 : 342 = 392 : x$, when $x = 275.854$ kilograms maltose. Ans.

$486 : 162 = 392 : x$, when $x = 131.743$ kilograms dextrin.

(6) See Arts. **52–55.**

(7) See Art. **57.**

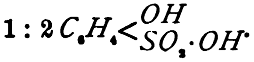
(8) The pyridine bases are found in coal tar, and in oil extracted from bones.

(9) Theobromine may be prepared synthetically from guanine. See Arts. **124** and **125.**

(10) See Art. **57.**

(11) See Art. **31.**

(12) Aseptol is phenol sulphonic acid; its formula is



$1:2\,C_6H_4<^{OH}_{SO_2\cdot OH}$

(13) See Art. **1.**

(14) Morphine has the formula $C_{17}H_{19}NO_3$.

(15) See Arts. **113–116.**

(16) See Art. **55.**

(17) See Arts. **28–30.**

(18) See Art. **87.**

(19) See Art. **128.**

(20) See Art. **63.**

(21) Phenol phthalein is employed as an indicator in volumetric analysis.

(22) See Art. **15.**

(23) See Art. **1.**

(24) Quinone is obtained from benzene, according to the equations given in Arts. **20** and **21.** The amount is obtained from the molecular weight of benzene (78) and that of quinone (108) by a simple proportion:

$108 : 78 = 134.306 : x$, when $x = 97$ grams. Ans.

(25) (*a*) and (*b*) See Art. **138.**

(26) Thiophene (see Art. **104**).

(27) See Art. **55.**

(28) No; rather an advantage, as these two compounds possess a very brilliant color, which will increase the brilliancy of that of alizarin.

(29) See Arts. **7, 8,** and **9.**

(30) See Art. **135.**

(31) See Art. **95.**

(32) Glucose is dextrorotary, while levulose is levorotary.

(33) See Art. **23.**

(34) Trinitrophenol, or picric acid.

(35) See Art. **136.**

(36) Disaccharides are characterized by being converted by hydrolysis into 2 molecules of glucose.

(37) (*a*) and (*b*) See Arts. **25** and **26.**

(38) The enlargement of the pupil of the eye.

(39) See Art. **92.**

(40) (*a*) In all carbohydrates oxygen and hydrogen are present in the proportion of 1 : 2. (*b*) Their general formula is $C_m(H_2O)_n$.

(41) See Art. **19.**

(42) A few drops of a ferric solution added to a solution suspected to contain phenol gives a beautiful blue color if that compound is present. Bromine water gives a yellowish-white precipitate of dibromophenol under similar conditions; and a pine shaving moistened with concentrate hydrochloric acid assumes a blue color after being dipped in a solution containing phenol and then exposed to the atmosphere.

(43) Nicotine occurs principally in the leaves of the tobacco plant.

(44) See Art. **86.**

(45) Glucose has the formula $C_6H_{12}O_6$, and a molecular weight of 180. Using formula **1,** *Theoretical Chemistry,*

$$x = \frac{100\,a\,n}{m},$$

we obtain $x = \frac{100 \times 12 \times 6}{180} = 40\%$ carbon;

$$x = \frac{100 \times 1 \times 12}{180} = 6.67\% \text{ hydrogen};$$

$$x = \frac{100 \times 16 \times 6}{180} = 53.33\% \text{ oxygen.} \quad \text{Ans.}$$

(46) See Art. **17.**

(47) $C_6H_5 \cdot OH + 3HNO_3 = C_6H_2(NO_2)_3OH + 3H_2O.$

(48) The alkaloids are usually classified as "volatile alkaloids" and "non-volatile alkaloids."

(49) No; most vegetable coloring matter does not resist sufficiently the decomposing action of light, air, etc., and consequently its color fades sooner than coloring matter obtained from other sources.

(50) Stereo-isomerides are compounds having the same percentage composition, but possessing different optical activity.

(51) The method to prepare ether analogous to that of methyl-phenol ether, is given in Art. **165,** *Organic Chemistry,* Part 2. The reaction that takes place may be expressed by the equation:

$$C_2H_5I + C_2H_5ONa = (C_2H_5)_2 : O + NaI$$

(52) The alkaloids are distinguished by being more or less poisonous; they are the "active principle" of poisonous plants.

(53) Pyrogallol is used in gas analysis, as it possesses the property to absorb oxygen.

(54) See Arts. **2** and **3.**

(55) Oxyhemoglobin occurs in the red globules of the arterial blood.

(56) See Art. **67.**

QUANTITATIVE ANALYSIS.

(PART 1.)

(1) See Art. **1.**

(2) Sodium hydrate, or sodium carbonate.

(3) (*a*) Magnesium-ammonium arsenate $MgNH_4AsO_4$.

(*b*) When this precipitate is heated, water and ammonia are driven off, and magnesium pyroarsenate $Mg_2As_2O_7$ is formed.

(4) See Art. **100.**

(5) $1.036 \times 24.73 \div 100 = .2562028$ weight of chlorine. $.2562028 \div .423 \times 100 = 60.57$ per cent. of Cl in sample. As dividing by 100 and then multiplying the result by 100 merely makes extra work without changing the final result, this is always omitted in actual work. This may be stated as follows:

$$\frac{1.036 \times 24.73}{.423} = 60.57. \quad \text{Ans.}$$

(6) (*a*) and (*b*) See Art. **3.**

(7) See Art. **21.**

(8) (*a*) A mixture of antimony sulphide Sb_2S_3, free sulphur, and water.

(*b*) See Art. **60.**

(9) See Art. **100.**

(10) $.250 \times 70 = 17.50$. $17.50 \div 1 = 17.50$ per cent. of iron found. Taking 56 for the atomic weight of iron and

278 for the molecular weight of ferrous sulphate, we have: $(56 \div 278) \times 100 = 20.14$ per cent. of iron calculated. $20.14 - 17.50 = 2.64$ per cent. error.

(11) In answering this question the student should give, in his own words, a brief outline of the general method of determining substances gravimetrically, from what he has learned in the Instruction Paper and by practice.

(12) As oxide or as metallic nickel. The metallic nickel may be obtained either by precipitation and reduction, or by electrolytic deposition. See also Arts. **22** and **24.**

(13) (*a*) and (*b*) See Arts. **58** and **59.**

(14) See "Volhard's Method for *Cl*, *Br*, *I*, *Ag*, and *Cu*."

(15) See Art. **6.**

(16) (*a*) CuS.

(*b*) Cu_2S.

(17) See Art. **71.**

(18) (*a*) and (*b*) See Arts. **38, 39, 40, 41,** and **100.**

(19) One cubic centimeter of decinormal silver nitrate precipitates .003537 gram of chlorine, which is equivalent to .005837 gram of sodium chloride. Hence,

$.005837 \times 30 = .17511$ gram of sodium chloride. Ans.

(20) See Art. **6.**

(21) (*a*) Magnesium-ammonium phosphate $MgNH_4PO_4$.

(*b*) When ignited, ammonia and water are expelled, and magnesium pyrophosphate $Mg_2P_2O_7$ is formed.

(22) By means of certain organic compounds, known as indicators, which change color when the reaction of the solution changes. See also Art. **78.**

(23) See Art. **101.**

(24) (*a*) and (*b*) See Art. **7.**

(25) (*a*) and (*b*) See Art. **39.**

(26) See Art. **66.**

(27) If a sample were not dried before making a determination, the result obtained would not represent the percentage of the given element in the compound analyzed, but would represent the percentage of this element in a certain mixture of this compound and water.

(28) See Art. **9.**

(29) (*a*) and (*b*) See Arts. **62** and **63.**

(30) (*a*) and (*b*) See Art. **35.**

(31) See Art. **111.**

(32) $NaCl + AgNO_3 = AgCl + NaNO_3$.

(33) (*a*) and (*b*) See Arts. **64** and **65.**

(34) See Art. **43.**

(35) See Art. **57.**

(36) The weights would bear the same relation to each other as the molecular weights of the compounds; hence, taking the molecular weights of copper sulphate and copper oxide as 249 and 79, respectively, we have

$$249 : 79 = 1 : x. \quad x = .3172. \quad \text{Ans.}$$

(37) See Art. **13.**

(38) Precipitates are heated in order to get them in forms in which their exact composition is known. In every case the heat serves to expel water, and in some cases it changes the precipitate from a compound of variable composition to a weighable compound whose exact composition is known. For instance, when ferric hydrate is heated, it is changed to ferric oxide, and when magnesium-ammonium phosphate is ignited, it is changed to magnesium pyrophosphate, etc.

(39) (*a*) and (*b*) See Art. **43.**

(40) (*a*) and (*b*) See Arts. **71** and **87.**

(41) This relation depends on the facts that the atomic weight of sulphur is twice that of oxygen, and that there are 2 atoms of copper united with 1 atom of sulphur, while there is 1 atom of copper united with 1 atom of oxygen;

hence, the metal and non-metal are united in the same proportion in each case. In the case of cuprous sulphide, we have 126 parts of copper united with 32 parts of sulphur, while in copper oxide we have 63 parts of copper united with 16 parts of oxygen. Multiplying the weights of copper and oxygen in this compound by 2, we have the metal and non-metal united in the ratio of 126 to 32 in each case.

(42) (*a*) As 20 cubic centimeters of normal sulphuric acid neutralize 20 cubic centimeters of normal sodium hydrate, this solution contains the same weight of sodium hydrate that would be contained in 20 cubic centimeters of a normal solution; hence, it is only necessary to calculate what this weight is. As 1 cubic centimeter of normal sodium hydrate contains .04 gram of the solid, 20 cubic centimeters contain .8 gram.

(*b*) Similarly for sodium. According to the definition of a normal solution; 1 liter contains 23 grams of sodium. Then 1 cubic centimeter contains .023 gram, and 20 cubic centimeters contain .46 gram of Na.

(43) The nitric acid is added to dissolve the silver, and the hydrochloric acid to precipitate it as chloride, thus replacing the chlorine that has been driven off. See also Art. **12.**

(44) (*a*) See Art. **68.**
(*b*) See Art. **34.**

(45) See Art. **90.**

(46) (*a*) See Art. **18.**
(*b*) See Art. **19.**

(47) See Art. **14.**

(48) (*a*) and (*b*) See Art. **55.**

(49) Methyl-orange.

(50) The oxide method, the sulphide method, and the electrolytic method.

(51) See "Acidimetry and Alkalimetry."

(52) See Art. **57.**

(53) (*a*) and (*b*) See Arts. **94** to **99.**

(54) As a molecule of cuprous sulphide contains 2 atoms of copper, and a molecule of copper sulphate contains but 1, 2 molecules of copper sulphate are required to produce 1 molecule of cuprous sulphide. Bearing this in mind, and taking the molecular weights of copper sulphate and cuprous sulphide as 249 and 158, respectively, we have

$$158 : 498 = .3 : x. \quad x = .9456 \text{ gram. Ans.}$$

(55) See Art. **91.**

(56) See Art. **19.**

(57) See Art. **15.**

(58) (*a*) and (*b*) See Arts. **24** and **25.**

(59) If the solution from which chlorine is to be precipitated is heated, chlorine will be expelled by the nitric acid added with the reagent, and the result will be too low. Silver is not volatilized in this way, and if some of the chlorine of the reagent is driven off, it makes no difference, as enough will be added to precipitate all the silver at any rate.

(60) See Art. **35.**

(61) See Arts. **26** and **27.**

(62) See Art. **94.**

(63) Cuprous sulphocyanide $Cu_2(SCN)_2$.

(64) See Art. **76.**

(65), (66), (67), (68), (69), (70) The percentage of the elements to be determined in the compounds sent for analysis cannot be given, as it will vary somewhat from time to time. A careful record of the composition of each sample is kept, however, and the student will be graded upon his results.

QUANTITATIVE ANALYSIS.

(PART 2.)

(1) See Art. **2.**

(2) CaO = 56 per cent. CO_2 = 44 per cent. See also Art. **21.**

(3) (*a*) The precipitate of antimony sulphide always contains more or less free sulphur and water. These must be expelled from a part of the precipitate after it is weighed, and from the amount of pure antimony compound obtained from the portion of precipitate taken, the amount of antimony sulphide or antimony in the precipitate is calculated. The sulphur and water may be driven off by heating a portion of the precipitate in an atmosphere of carbon dioxide, or a portion of the precipitate may be treated with fuming nitric acid, and the antimony weighed as oxide, after driving off the acid mixture by heat.

(*b*) If a correction were not made by one of the methods given, the water and free sulphur would be weighed as antimony sulphide, and an erroneous result would thus be obtained.

(4) See Art. **121.**

(5) See Art. **3.**

(6) See Art. **22.**

(7) See Arts. **64, 65,** and **66.**

(8) See Art. **132.**

(9) See Art. **4.**

(10) See Art. **30.**

(11) (*a*) See Art. **49.**
(*b*) See Art. **51.**

(12) Fe_2O_3 contains 90 per cent. of FeO; hence, $.225 \times .90 = .2025$ gram of FeO, and as the weight of a constituent divided by the weight of the original substance, and this number multiplied by 100, gives the percentage of that constituent in the substance, $.2025 \div 1 = .2025$. $.2025 \times 100 = 20.25$ per cent. of FeO. Ans.

(13) (*a*) and (*b*) See Art. **5.**

(14) (*a*) and (*b*) See Art. **31.**

(15) Now no method has been given in the text for the analysis of an alloy containing only these two metals, but methods of separating and determining them are given in the analysis of nickel coins (Art. **59,** *et seq.*) and in the analysis of chalcopyrite, and one of these methods may be employed, omitting, of course, the portions referring to other elements. See Arts. **115** and **116.**

(16) As the stone contains 90 per cent. of calcium carbonate, it contains $2,000 \times .90 = 1,800$ pounds of calcium carbonate; and as calcium carbonate contains 56 per cent. of calcium oxide, $1,800 \times .56 = 1,008$ pounds of lime. Ans.

(17) (*a*) and (*b*) See Art. **6.**

(18) See Arts. **32** to **37,** inclusive.

(19) See Art. **87.**

(20) As ZnO contains 80.26 per cent. of zinc,

$.1505 \times .8026 = .1207913$ gram of zinc.
$.1207913 \div .625 = .1933.$
$.1933 \times 100 = 19.33$ per cent. of zinc. Ans.

(21) A porcelain crucible with a perforated bottom similar to a Gooch crucible. This is frequently called a porcelain Gooch crucible. See also Art. **6.**

(22) See Art. **39,** *et seq.*

(23) See Art. **88,** *et seq.*

(24) Feldspar is essentially a silicate of aluminum and potassium with a smaller quantity of sodium, or of aluminum and sodium with a smaller quantity of potassium, and in addition to these constituents, it nearly always contains smaller amounts of iron, calcium, and magnesium. See also Art. **139.**

(25) (*a*) and (*b*) See Arts. **6** and **11.**

(26) See Arts. **40** and **42.**

(27) See Art. **92.**

(28) See Art. **7.**

(29) (*a*) See Art. **43.**
(*b*) See Art. **44,** *et seq.*

(30) They may be separated by fusing the precipitate of iron and alumina with acid potassium sulphate and proceeding as directed in Art. **141,** but it is better to take a fresh sample and proceed as directed in Art. **95.**

(31) See Art. **12.**

(32) See Art. **58,** *et seq.*

(33) See Art. **20.**

(34) See Art. **48.**

(35) (*a*) and (*b*) See Art. **96.**

(36) See Art. **144.**

(37) See Arts. **11** and **12.**

(38) (*a*) It is an alloy of lead and antimony, containing the metals in varying proportions, the percentage of each metal depending upon the kind of type for which it is intended.

(*b*) It is dissolved for analysis by treating a sample of the finely divided alloy with a mixture of equal volumes of concentrate nitric and tartaric acids. See also Art. **54.**

(39) See Art. **11.**

(40) (*a*) and (*b*) See Art. **58.**

(41) See Art. **12.**

(42) (*a*) The iron is weighed in the form of ferric oxide Fe_2O_3.

(*b*) As the iron in the original substance is in the ferrous state, it is reported as ferrous oxide FeO. See also Arts. **17** and **19.**

(43) See Art. **55.**

(44) (*a*) and (*b*) See Art. **109.**

(45) As the limestone contains 40 per cent. of carbon dioxide, 1 gram of it would contain 40 per cent. of 1 gram, or .4 gram of carbon dioxide. 1,000 cubic centimeters (1 liter) of carbon dioxide weighs 1.97 grams; hence,

$1.97 : .4 = 1{,}000 : x$. $x = 203.05$ cubic centimeters. Ans.

(46) As Fe_2O_3 contains 90 per cent. of FeO, the weight of FeO may be obtained by multiplying the weight of Fe_2O_3 by .90; or it may be calculated by a proportion. As 1 molecule of Fe_2O_3 contains 2 molecules of FeO, the proportion would be

$$Fe_2O_3 : 2FeO = \text{wt. of } Fe_2O_3 : x.$$

(47) It is composed of tin, lead, bismuth, and cadmium.

(48) See Arts. **104** to **107,** inclusive.

(49) See Art. **19.**

(50) No scheme is given for the analysis of an alloy containing only these two metals, but a method of separating and determining them is given in the analysis of Wood's metal (Art. **73,** *et seq.*), and the method here given may be employed by omitting the portions referring to other metals.

(51) See Art. **114.**

(52) (*a*) Zinc is present in the solution in the form of chloride, and as it is precipitated as sulphide, the chlorine set free unites with the hydrogen of the hydrogen sulphide,

forming hydrochloric acid. Zinc sulphide is soluble in a solution containing any considerable amount of free hydrochloric acid, and enough acid may be formed during the reaction to prevent the complete precipitation of the zinc. If sodium acetate is now added, the hydrochloric acid unites with the sodium, forming sodium chloride, and setting free acetic acid, in which zinc sulphide is insoluble, and consequently the zinc will be completely precipitated.

(*b*) If enough sodium acetate is added to unite with all the hydrochloric acid, the nickel and cobalt will also be precipitated; hence, care must be taken to leave enough free hydrochloric acid in the solution to prevent the precipitation of these metals. Enough hydrochloric acid to accomplish this may be left in the solution without interfering with the precipitation of zinc.

(53) See Art. **16.**

(54) As the alloy contains 17 per cent. of antimony, the weight of antimony required will be 17 per cent. of the weight of the alloy, or $125 \times .17 = 21.25$ pounds. Ans.

(55) See Art. **19.**

(56) See Art. **78.**

(57) See Art. **10.**

(58) See Art. **109,** *et seq.*

(59) As the sample contains 90 per cent. of silver, and .8 gram of it are taken for analysis, we have $.8 \times .90 = .72$ gram of silver in the sample. As this is all converted into chloride, taking the atomic weights of silver and chlorine as 107.66 and 35.37, respectively, we have

$$107.66 : 143.03 = .72 : x. \quad x = .9565 \text{ gram. Ans.}$$

(60) See Art. **21.**

(61) (*a*) Bismuth oxychloride, basic bismuth nitrate, and basic bismuth carbonate.

(*b*) Bismuth oxychloride $BiOCl$, bismuth oxide Bi_2O_3, and metallic bismuth.

(62) As Cu_2S contains 79.82 per cent. of copper, we have $.952 \times .7982 = .759886$ gram of copper in the sample; and as the bronze contains 92 per cent. of copper, this is 92 per cent. of the weight of the sample. Hence,

$.759886 \div .92 = .8295$ gram of sample taken. Ans.

(63), (64), (65), (66), (67), (68), (69), (70) The percentage of the elements to be determined in the compounds sent for analysis cannot be given, as it will vary somewhat from time to time. A careful record of the composition of each sample is kept, however, and the student will be graded upon his results.

QUANTITATIVE ANALYSIS.

(PART 3.)

(1) The value of an iron ore depends on the amount of iron it contains and its freedom from impurities.

(2) The elements usually determined in pig iron are silicon, sulphur, phosphorus, and manganese. Carbon is also determined sometimes, but not as a rule.

(3) An acid solution of the double chloride of copper and potassium. For its preparation see Art. **60.**

(4) See Art. **80.**

(5) A zinc-copper couple is prepared by immersing zinc in a copper-sulphate solution that contains about 3 per cent. of the crystallized salt. See also Art. **114.**

(6) An iron ore is analyzed to determine its value, its fitness for a certain purpose, and the amount of other material that must be charged into the furnace with the ore.

(7) See Art. **28.**

(8) (*a*) Silver sulphate is used to purify the carbon dioxide evolved, by absorbing any hydrochloric acid it may contain.

(*b*) For its preparation see Art. **63.**

(9) Water is most frequently examined to determine its fitness for use either for domestic purposes or as a boiler supply.

(10) See Arts. **114** and **115.**

(11) The determinations generally made in the analysis of iron ores are insoluble matter and silica, iron, phosphorus, sulphur, and manganese. Water is also frequently determined.

(12) See Art. **28.**

(13) See Art. **67.**

(14) Multiply the number of parts per million by 58,318 and divide the result by 1,000,000, to obtain the grains per U. S. gallon. See also Art. **81.**

(15) Copper, lead, and zinc are probably the poisonous metals most frequently found in water in sufficient quantity to be dangerous. Iron probably occurs in water more frequently than any of these, but usually in insignificant quantities. Arsenic and chromium are also occasionally found in water, and both are very poisonous.

(16) See Art. **3.**

(17) The sample should be transferred from one paper to another by means of a magnet, thus leaving the sand and scale, which are non-magnetic, on the first paper. For the details of the process see Art. **28.**

(18) See Art. **67.**

(19) The principal determinations made in the examination of potable waters are total solids, chlorine, free and albuminoid ammonia, oxygen consumed, nitrogen as nitrite, nitrogen as nitrate, and poisonous metals. The hardness is also determined sometimes, but, as it has little to do with the quality of a drinking water, this determination is frequently omitted.

(20) See Art. **118.**

(21) See Art. **3.**

(22) See Art. **30** *et seq.*

(23) As coke is composed of fixed carbon and ash, the percentage of coke in the sample is obtained by adding

the percentages of these constitutents; hence, this sample contains 60.25 per cent. of coke. As there are 5,000 pounds of the coal containing 60.25 per cent. of coke, we have 5,000 × .6025 = 3,012.5 pounds coke. Ans.

(24) See Art. **83.**

(25) See Art. **125.**

(26) (*a*) When a cast-iron mortar and pestle are used to pulverize iron ores, the iron wears off, especially from the pestle, and, becoming mixed with the ore, gives it a fictitious value.

(*b*) A mortar and pestle of hardened steel, or a bucking board and muller of chilled iron, should be used for this purpose.

(27) The starch solution is used as an indicator. As soon as the iodine has broken up all the hydrogen sulphide, with the formation of hydriodic acid, it begins to unite with the starch, forming blue starch iodide, and thus indicating that the reaction is complete.

(28) See Art. **69.**

(29) See Art. **86.**

(30) The principal scale-forming constituents in water are the carbonates of calcium and magnesium, and calcium sulphate.

(31) Clay is a mixture of silica with the hydrated silicates of aluminum calcium, magnesium, sodium, potassium, and generally a small amount of iron. See also Art. **72.**

(32) See Art. **35.**

(33) (*a*) See Art. **71.**
(*b*) See Art. **69.**

(34) The method employed for the determination of chlorine in water depends on the relative affinity of silver for chlorine and chromic acid. When silver nitrate is added to water containing both a chloride and a chromate, the silver unites with chlorine until all the chlorine is combined with

silver. Any silver added after this, unites with the chromic acid, forming red silver chromate. Consequently, when we wish to determine chlorine in water, we add a chromate and lead in silver nitrate. White silver chloride is formed at first, but, after a time, a red color is developed, showing that enough silver has been added to unite with all the chlorine, and silver chromate is beginning to form. Then, knowing the strength of the silver solution and the amount used, the amount of chlorine is readily calculated.

(35) If the moisture is to be determined in an ore, or if the composition of the ore as it is purchased, or charged into the furnace, is wanted, the sample should be pulverized quickly, for a damp ore will lose moisture, and a very dry one may absorb it from the air during this operation.

(36) See Art. **5.**

(37) (*a*) See Art. **48.**

(*b*) The chief advantages of this method are its simplicity, and the rapidity with which it yields results.

(38) (*a*) Iron and the alkalies are injurious on account of the relatively low temperature at which they fuse.

(*b*) A first-class clay for this purpose should not contain more than 2 per cent. of ferric oxide, or more than 1 per cent. of either alkali.

(39) See Art. **88.**

(40) (*a*) See Art. **5.**

(*b*) It is used at first when the ore is partly pulverized, to separate the coarse portion from the fine, and, later, to enable us, by passing it all through the sieve, to be sure that it is all fine enough for analysis.

(41) See Arts. **6, 7, 8,** and **9.**

(42) See Art. **50.**

(43) The constituents of clay usually determined are silica, alumina, ferric oxide, calcium oxide, magnesia, and the alkalies.

(44) See Arts. **92** and **93.**

(45) See Art. **52.**

(46) See Art. **10.**

(47) (*a*) Clay usually contains from 6 to 14 per cent. of combined water.

(*b*) For its determination see Art. **74.**

(48) Albuminoid ammonia does not exist in water in the form of ammonia, but is in the form of nitrogenous organic matter, which is broken up when boiled with potassium hydrate and potassium permanganate, yielding an amount of ammonia proportional to the amount of nitrogenous matter in the water.

(49) (*a*) and (*b*) See Art. **15.**

(50) $2FeCl_3 + SnCl_2 = 2FeCl_2 + SnCl_4$.

(51) See Art. **54.**

(52) (*a*) and (*b*) See Art. **95.**

(53) In Art. **27,** it is stated that the value of a permanganate solution in manganese is 29.46 per cent. of its value in iron. The value of 1 cubic centimeter of this solution in iron is .01 gram, and its value in manganese is, therefore, 29.46 per cent. of .01 or .002946 gram. As 5.5 cubic centimeters of permanganate are used, we have $.002946 \times 5.5 = .0162$ gram of manganese in the solution; and, as 1.5 grams of sample are dissolved, and two-thirds of the solution (= 1 gram of sample) are titrated, we find that 1 gram of the sample contains .0162 gram, or the sample contains 1.62 per cent. of manganese. Ans.

(54) (*a*) Potassium sulphocyanide is used to test for ferric iron, in order to be sure that the iron is completely reduced.

(*b*) The solution for this purpose is prepared by dissolving from 5 to 10 grams of the solid in 100 cubic centimeters of distilled water.

(55) See Art. **55.**

(56) See Art. **97.**

(57) $H_2S + I_2 = 2HI + S.$

(58) (*a*) Potassium ferricyanide is used as an indicator when an iron solution is titrated with potassium bichromate, to show when the iron is completely oxidized.

(*b*) The solution should always be tested for ferrocyanide when made up, and a fresh solution should be prepared every day or two.

(59) See Art. **58.**

(60) The apparatus used in water analysis should be cleaned by rinsing it thoroughly with water after each analysis, and then rinsing it again before another analysis is made. Good tap water is usually sufficiently pure for this purpose, but if the water supply is not of good quality, the apparatus should finally be rinsed out with distilled water. Apparatus used for this purpose should never be wiped with a cloth, but should be allowed to drain.

(61)

$$K_2Cr_2O_7 + 6KI + 14HCl = 8KCl + 2CrCl_3 + 7H_2O + 3I_2.$$

(62) See Art. **59.**

(63) See Art. **110.**

(64) When a pulverized sample of coal is allowed to stand in the air, it will give up moisture if damp, and, if dry, it will absorb moisture from the atmosphere. In addition to this, the composition of the sample itself will change; hence, the powdered sample should be kept in a tightly stoppered bottle, and analyzed as soon as possible.

(65)

$$2KMnO_4 + 3MnSO_4 + 2H_2O = 5MnO_2 + 2KHSO_4 + H_2SO_4.$$

(66) When naphthyl amine is added to water containing nitrites, it produces a pink color similar to that of a dilute permanganate solution.

(67) Organic matter, especially animal matter, contains considerable nitrogen, and, as it decomposes in the water, the nitrogen is partially oxidized, forming nitrites. It is thought that the nitrites in drinking water are generally formed in this way. Nitrites may come from other sources, but this is believed to be the chief source.

(68) See Art. **61.**

(69) See Art. **113.**

(70) See Art. **43.**

QUANTITATIVE ANALYSIS.

(PART 4.)

(1) Using formula **10,**

$$C = \frac{100\,p - 1,350}{86.5},$$

and substituting the known values, we obtain

$$C = \frac{100 \times 15 - 1,350}{86.5} = 1.734\%\text{ ceresin in sample.}\quad\text{Ans.}$$

(2) By "potential ammonia" is meant the nitrogen contained in the fertilizer, that, by the progress of decomposition of the organic matter of the fertilizer, will be transformed into ammonia.

(3) $$\frac{11.2 \times 273 \times 771}{(273 + 22) \times 760} = 10.52+\text{ liters.}\quad\text{Ans.}$$

(4) See Art. **238.**

(5) As 1 cubic centimeter of the arsenious solution is equivalent to .0033 gram of chlorine, 47.4 cubic centimeters are equivalent to $47.4 \times .0033 = .15642$ gram in 50 cubic centimeters of the solution; but 50 cubic centimeters contain .575 gram of bleaching powder, hence the substance contains

$$\frac{.15642 \times 100}{.575} = 27.2\%\text{ of available chlorine.}\quad\text{Ans.}$$

(6) See Art. **29.**

(7) See Art. **206.**

(8) $C_{12}H_{22}O_{11}+H_2O = C_6H_{12}O_6+C_6H_{12}O_6$. Cane sugar is transformed by inversion into glucose and fructose, as is seen by above equation.

(9) Subacetate of lead, prepared as described in Art. **169,** is most generally used.

(10) See Art. **120.**

(11) See Art. **16.**

(12) See Art. **36** *et seq.*

(13) See Art. **2.**

(14) Using formula **9,**

$$P = 100-\frac{100\ S}{92.8},$$

and substituting the known values, we obtain

$$P = 100-\frac{100\times 87.6}{92.8} = 5.6+,$$

which is the amount of paraffin present in the sample.

(15) The percentage of iodine absorbed equals

$$\frac{26.1\times .0122\times 100}{1.0214} = 31.174,$$

which would be the iodine number of the fat under examination.

(16) See Art. **218.**

(17) Using formula **7,**

$$R = \frac{100\,S}{142.66-13.5},$$

and substituting the known values, we obtain

$$R = \frac{100\times(86+19)}{142.66-13.5} = 81.29\% \text{ of cane sugar. Ans.}$$

(18) Soluble or water-soluble phosphoric acid; citrate-soluble phosphoric acid; citrate-insoluble phosphoric acid.

(19) No. See Art. **81.**

(20) See Art. **29.**

(21) See Art. **9.**

(22) See Arts. **230** and **231.**

(23) The iodine number in fat determination represents the quantity of iodine that a fat is able to absorb, and thus serves as a measure of the unsaturated acids present in the fat.

(24) Using formula **7,**

$$R = \frac{100\,S}{142.66 - \frac{1}{2}t^\circ},$$

and substituting the known values, we obtain

$$R = \frac{100 \times (93 + 21)}{142.66 - 10.5} = 86.259\% \text{ of cane sugar.} \quad \text{Ans.}$$

(25) 123.979 milligrams K_2O.

(26) Specific gravity, total solids, ash, total proteids, casein, albumin, and sugar.

(27) Using formula **5,**

$$W = \frac{100\,v(P-p)}{760 \times 354.33\,a \times (1 + .00366\,t)},$$

and substituting the known values, we obtain

$$\frac{100 \times 12 \times (767 - 11.2)}{760 \times (354.33 \times 2) \times (1 + .00366 \times 13)} = 1.607 \text{ grams;}$$

and as the specific gravity of the sample is 1.021, 100 cubic centimeters weigh 102.1 grams and the percentage of urea is

$$\frac{1.607 \times 100}{102.1} = 1.573. \quad \text{Ans.}$$

(28) Using formula **4,**

$$V_0 = \frac{V(P-p) \times 273}{760(273 + t)},$$

and substituting the known values, we obtain

$$V_0 = \frac{97(765 - 10.5) \times 273}{760 \times (273 + 12)} = 92.24+ \text{ cubic centimeters.} \quad \text{Ans.}$$

(29) See Arts. **204, 205,** and **206.**

(30) The saponification number indicates the number of milligrams of potassium hydrates required to saponify 1 gram of the fatty substance, and, therefore, represents the capacity of saturation of the fatty acids contained in the sample.

(31) See Art. **179.**

(32)

Sugar	95.00%
Water	2.20%
Ashes	1.32%
Foreign organic substance	1.48%
Total	100.00%

(33) See Art. **122.**

(34) The specific gravity of pure milk varies from 1.208 to 1.035; its average is about 1.0319—.

(35) See Art. **58.**

(36) $\frac{40.72 \times 273 \times 730}{(273+12) \times 760} = 37.466$ cubic centimeters. .1 liter of nitrogen at normal temperature and pressure weighs 1.257 grams, hence, 37.466 cubic centimeters weigh 470.95 milligrams, which equals 4.639 per cent. Ans.

(37) The calculation is similar to that explained in Art. **189.** The values for $Cu = .385$, $W = 4.250$, and the percentage of cane sugar, 89 per cent., are known. Before using formula **8,** we must calculate the value for Z and the proportion of $R : I$, in order to be able to find the value for F in the formula.

$$\frac{Cu}{2} = \frac{.385}{2} = .1925 = Z.$$

$$Z \times \frac{100}{W} = .1925 \times \frac{100}{4.25} = 4.529 = y.$$

$$\frac{100\,P}{P+y} = \frac{8{,}900}{89+4.529} = 95.15 = R.$$

$$100 - R = 100 - 95.15 = 4.85 = I.$$

Then,

$$R : I = 95.15 : 4.85.$$

By consulting Table 5, we find that the vertical column $I = 200$ is nearest to Z (.1925), the horizontal column headed 95 : 5, is nearest to the ratio $R : I = 95.15 : 4.85$. Where these two columns meet, we find the factor 52.6, and we can now make the final calculation by substituting the known values in formula **8.**

$\frac{Cu\,F}{W} = \frac{.385 \times 52.6}{4.250} = 4.764\%$ of invert sugar in sample under examination. Ans.

(38) See Art. **183.**

(39) 26.048 grams pure cane sugar.

(40) See Art. **129.**

(41) See Art. **116.**

(42) See Art. **50.**

(43) See Art. **11.**

(44) See Art. **227.**

(45) See Art. **181.**

(46) See Arts. **176, 177,** and **178.**

(47) 96.029 milligrams PCl.

(48) See Art. **93.**

(49) For the preparation of Fehling's solution, see Art. **78,** *Qualitative Analysis*, Part 2; 1 cubic centimeter of this solution is equivalent to .005 milligram of sugar.

(50) As 50 cubic centimeters of gas are taken, and this, on average, contains 20 per cent. CO_2, the amount of CO_2 in each determination weighs 19.77 milligrams. The absorption of CO_2 by a potassium-hydrate solution is expressed by the equation: $2KOH + CO_2 = K_2CO_3 + H_2O$ and 43.89 parts of CO_2 are absorbed by 112 parts KOH. Then, $43.89 : 112 = 19.77 : x$, when $x = 50.45$ milligrams KOH, and as 600 determinations are made, 50.45 milligrams × 600 = 30.27 grams, or 30.27 grams of KOH would be consumed.

(51) The first step to be taken is to calculate the percentage of cane sugar; we employ formula **7**,

$$R = \frac{100\,S}{142.66 - \frac{1}{2}t^{\circ}},$$

and substitute the known values,

$$R = \frac{100 \times (69 + 24)}{142.66 - 11} = 70.636\% \text{ cane sugar.} \quad \text{Ans.}$$

For the calculation of the invert sugar, we have to calculate the proportion $R : I$, and the value for Z, in order to obtain the value F, which is needed for formula **8.**

The known values are $Cu = .315$, $W = 3.5$, and the percentage of cane sugar $P = 70.636$ per cent.

$$\frac{Cu}{2} = \frac{.315}{2} = .1575 = Z.$$

$$Z \times \frac{100}{W} = .1575 \times \frac{100}{3.5} = 4.5 = y.$$

$$\frac{100\,P}{P + y} = \frac{7{,}063.6}{70.636 + 4.5} = 94.01 = R.$$

$$100 - R = 100 - 94.01 = 5.99 = I.$$

$$R : I = 94.01 : 5.99.$$

By consulting Table 5, we find that the vertical column headed "$I = 150$" is nearest to Z ($= .1575$), and the vertical column 94 : 6 is nearest to the ratio $R : I$ (94.01 : 5.99). Where these two columns meet, we find the factor 51.6, and can now apply formula **8**,

$$\frac{Cu\,F}{W} = \frac{.315 \times 51.6}{3.5} = 4.644\% \text{ invert sugar.}$$

$$\text{Hence, the sample contains } \left.\begin{array}{l} 70.636\% \text{ cane sugar.} \\ 4.644\% \text{ invert sugar.} \end{array}\right\} \text{Ans.}$$

(52) Since 1 cubic centimeter of standard potassium permanganate corresponds to .004 gram of iron, and 24.6 cubic centimeters correspond to .0984 gram, then, .323 gram of iron taken, less .0984 gram of iron not oxidized by the bleaching powder, equals .2246 gram of iron oxidized by the sample.

Since 56 parts of Fe correspond to 35.5 parts of Cl, we obtain the proportion:

$$56 : 35.5 = .2246 : x,$$

when $x = .1424$ gram available Cl. .4 gram bleaching powder contains .1424 available chlorine. 1 gram contains .356 gram, or 35.6 per cent. Ans.

(53) 58.647 milligrams K_2SO_4.

(54) Using formula **6,**

$$t = \left(\frac{100\ S - FS}{100 - 1.0753\,FS} - 1\right)(250 - 2.5\ F),$$

and substituting the known values, we obtain

$$t = \left[\frac{100 \times 1.0328 - (2.56 \times 1.0328)}{100 - 1.0753(2.56 \times 1.0328)} - 1\right][250 - (2.5 \times 2.56)],$$

when $t = 8.720$.

Then, total solids not fat = 8.720%, and fat = 2.56%; hence, total solids = 8.720 + 2.56 = 11.28%. Ans.

(55) See Arts. **59** and **60.**

(56) As 210 milligrams of cupric oxide equal 168 milligrams Cu, referring to Table 3, we find

$$\begin{array}{lcl} 165 \text{ milligrams } Cu & = & 120.20 \\ \underline{3} \text{ milligrams } Cu \times .75 & = & \underline{2.25} \\ 168 \text{ milligrams } Cu & = & 122.45 \text{ milligrams } C_{11}H_{22}, H_2O. \end{array}$$

Ans.

(57) See Art. **64** *et seq.*

(58) See Art. **114** *et seq.*

(59) See Art. **224.** 33.5 − 25 = 8.5; .000176 × 8.5 = .0015. Then, 1.4652 + .0015 = 1.4667 refractive index at standard temperature. Ans.

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